AP CALCULUS AB

POLYNOMIALS TIMES FUNCTIONS WHOSE DERIVATIVES REPEAT

1) Calculate

$$\frac{d^{447}f}{dx^{447}}$$

for

a) $f(x) = e^x$

b) $f(x) = \sinh x$

c) $f(x) = \sin x$

(a) $\frac{d^{447}e^x}{dx^{447}} = e^x$

(b) $\frac{d^{447}\sinh x}{dx^{447}} = \frac{d}{dx}\frac{d^{446}\sinh x}{dx^{446}} = \frac{d}{dx}\sinh x = \cosh x$

(c) $\frac{d^{447}\sin x}{dx^{447}} = \frac{d^3}{dx^3}\frac{d^{444}\sin x}{dx^{444}} = \frac{d^3\sin x}{dx^3} = -\cos x$

2) Evaluate

$$I = \int x^2 \cos x \, dx = \int f'(x)\, dx = f(x)$$

by assuming a function of the form

$$f(x) = Ax^2\cos x + Bx^2 \sin x + Cx\cos x + \\ + Dx\sin x + E\cos x + F\sin x,$$

differentiating it, and choosing the constants A, B, C, D, E and F accordingly.

$f'(x) = A(2x\cos x + x^2\cdot -\sin x) + B(2x\sin x + x^2\cos x) + C(\cos x + x\cdot -\sin x) +$

$+ D(\sin x + x\cos x) + E\cdot -\sin x + F\cos x =$

$= -Ax^2\sin x + Bx^2\cos x + (2B - C)x\sin x + (2A + D)x\cos x + (D - E)\sin x + (C + F)\cos x =$

$= x^2\cos x \Rightarrow A = 0 \quad B = 1 \quad C = 2 \quad D = 0 \quad E = 0 \quad F = -2$ so

$I = f(x) = x^2\sin x + 2x\cos x - 2\sin x$