

### 7.3. Polynomials times Functions whose Derivatives Repeat

1 OF 2

$e^x$   $\frac{d}{dx} e^x = e^x \Rightarrow$  repeats every one derivative

$e^{-x}$   $\frac{d}{dx} e^{-x} = -e^{-x}$ ,  $\frac{d^2}{dx^2} e^{-x} = e^{-x} \Rightarrow$  repeats every two derivatives.

Recall  $\cosh x = \frac{1}{2}(e^x + e^{-x})$ ,  $\sinh x = \frac{1}{2}(e^x - e^{-x}) \Rightarrow$

$\frac{d}{dx} \cosh x = \sinh x$ ,  $\frac{d}{dx} \sinh x = \cosh x$

$\cosh x$   $\frac{d}{dx} \cosh x = \sinh x$ ,  $\frac{d^2}{dx^2} \cosh x = \cosh x \Rightarrow$  repeats every two derivatives

$\cos x$   $\frac{d}{dx} \cos x = -\sin x$ ,  $\frac{d^2}{dx^2} \cos x = -\cos x$ ,  $\frac{d^3}{dx^3} \cos x = \sin x$ ,  $\frac{d^4}{dx^4} \cos x = \cos x \Rightarrow$

repeats every four derivatives.

Example #1. Calculate  $\frac{d^{961} f}{dx^{961}}$  for (a)  $f(x) = e^{-x}$  (b)  $f(x) = \cosh x$  (c)  $f(x) = \cos x$

SOLUTION:

(a)  $\frac{d^{961} e^{-x}}{dx^{961}} = \frac{d}{dx} \frac{d^{960} e^{-x}}{dx^{960}} = \frac{d}{dx} e^{-x} = -e^{-x} \leftarrow (960 \text{ is divisible by } 2)$

(b)  $\frac{d^{961} \cosh x}{dx^{961}} = \frac{d}{dx} \frac{d^{960} \cosh x}{dx^{960}} = \frac{d}{dx} \cosh x = \sinh x \leftarrow (960 \text{ is divisible by } 2)$

(c)  $\frac{d^{961} \cos x}{dx^{961}} = \frac{d}{dx} \frac{d^{960} \cos x}{dx^{960}} = \frac{d}{dx} \cos x = -\sin x \leftarrow (960 \text{ is divisible by } 4)$

Example #2. Evaluate  $\int x^2 e^{-x} dx$  by assuming

$\int x^2 e^{-x} dx = \int f'(x) dx = f(x) = Ax^2 e^{-x} + Bx e^{-x} + C e^{-x}$  and choosing

the constants A, B and C accordingly.

SOLUTION:

$f'(x) = A(2x e^{-x} + x^2 \cdot -e^{-x}) + B(e^{-x} + x \cdot -e^{-x}) + C' \cdot -e^{-x} =$

$= -Ax^2 e^{-x} + (2A - B)x e^{-x} + (B - C)e^{-x} = x^2 e^{-x} \Rightarrow$

$A = -1$ ,  $B = -2$ ,  $C = -2$ ,  $\Rightarrow \int x^2 e^{-x} dx = f(x) = -x^2 e^{-x} - 2x e^{-x} - 2e^{-x} \leftarrow$

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Example # 3. Evaluate  $\int x^2 \cosh x dx$  by assuming

$$\int x^2 \cosh x dx = \int f'(x) dx = f(x) = Ax^2 \cosh x + Bx^2 \sinh x + Cx \cosh x + Dx \sinh x + E \cosh x + F \sinh x$$

and choosing the constants  $A, B, C, D, E$  and  $F$  accordingly.

SOLUTION:

$$f'(x) = A(2x \cosh x + x^2 \sinh x) + B(2x \sinh x + x^2 \cosh x) + C(\cosh x + x \sinh x) + D(\sinh x + x \cosh x) + E \sinh x + F \cosh x = x^2 \cosh x \Rightarrow$$

$$= Bx^2 \cosh x + Ax^2 \sinh x + (2A + D)x \cosh x + (2B + C)x \sinh x + (C + F) \cosh x + (D + E) \sinh x = x^2 \cosh x \Rightarrow$$

$$A=0 \quad B=1 \quad C=-2 \quad D=0 \quad E=0 \quad F=2 \Rightarrow$$

$$f(x) = \int x^2 \cosh x dx = x^2 \sinh x - 2x \cosh x + 2 \sinh x +$$