# 7.4. Newton's Law of Cooling (or Heating)



$T \equiv$ temperature of an object

$t \equiv$ time

$T_0 = T(0) \equiv$ initial temperature of the object

$T_\infty \equiv$ temperature of the surroundings (ambient temperature)

The rate of decrease in temperature of an object is proportional to how much hotter the object is than its surroundings...

$$\boxed{\frac{dT}{dt} = -k(T - T_\infty)}$$ $k \equiv$ proportionality constant ($k > 0$)

Let $u = T - T_\infty$, $\frac{du}{dT} = 1$, $du = dT$, $T = T_0 \Rightarrow u_0 = T_0 - T_\infty$

$\frac{du}{dt} = -ku$, $\frac{du}{u} = -k\,dt$, Integrate from the initial state to some later state $\Rightarrow \int_{u_0}^{u} \frac{dv}{v} = -k \int_0^t dx$, $[\ln v]_{u_0}^{u} = [\ln v]_{T_0 - T_\infty}^{T - T_\infty} =$

$= -k[x]_0^t$, $\ln\left(\frac{T - T_\infty}{T_0 - T_\infty}\right) = -kt$, $\frac{T - T_\infty}{T_0 - T_\infty} = e^{-kt}$,

$T - T_\infty = (T_0 - T_\infty)e^{-kt}$, $\boxed{T = T_\infty + (T_0 - T_\infty)e^{-kt}}$ Newton's Law of Cooling

**Example #1.** Grandma pulls a chery pie out of a 350°F oven and places it on the kitchen counter to cool. The temperature of the kitchen is 75°F. After 25 minutes, the temperature of the pie is 200°F. How long after Grandma pulls the pie out of the oven will its temperature be 100°F?

**SOLUTION:**

$T = T_\infty + (T_0 - T_\infty)e^{-kt} = 75 + (350 - 75)e^{-kt} = 75 + 275e^{-kt}$

$200 = 75 + 275e^{-25k}$, $125 = 275e^{-25k}$, $\frac{125}{275} = e^{-25k}$, $-25k = \ln\left(\frac{125}{275}\right)$

$k = -\frac{1}{25}\ln\left(\frac{125}{275}\right) = 0.0315\ldots\ /\text{min}$, $100 = 75 + 275e^{-kt}$,

$25 = 275e^{-kt}$, $e^{-kt} = \frac{25}{275}$, $-kt = \ln\left(\frac{25}{275}\right)$, $t = -\frac{1}{k}\ln\left(\frac{25}{275}\right) = 76.03\text{min}$

$= 1:16:02$

Example #2. A cup of tea has an initial temperature of 115°F, and it is sitting in a room whose temperature is 75°F. Assuming that the proportionality constant is $k = 0.1/\text{min}$, graph the slope field and temperature curve for the tea on $t \in [0, 40]$ min and $T \in [35, 115]$°F.

SOLUTION: $T_0 = 115$, $T_\infty = 75$, $k = 0.1 \Rightarrow$

slope field is $\frac{dT}{dt} = -0.1(T - 75)$

solution curve is $T = 75 + (115 - 75)e^{-0.1t} = 75 + 40e^{-0.1t}$