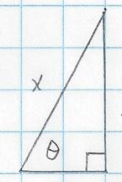
Quiz #12 Study Guide

(1) 

$\cos\theta = \frac{a}{x}$, $\sec\theta = \frac{x}{a}$, $x = a\sec\theta$, $\tan\theta = \frac{\sqrt{x^2-a^2}}{a}$,

$\sqrt{x^2-a^2} = a\tan\theta$, $\frac{dx}{d\theta} = a\sec\theta\tan\theta$, $dx = a\sec\theta\tan\theta\, d\theta$

$$\int \sqrt{x^2-a^2}\,dx = \int a\tan\theta \cdot a\sec\theta\tan\theta\, d\theta = a^2\int \sec\theta\tan^2\theta\, d\theta =$$

$$= \frac{a^2}{2}\sec\theta\tan\theta - \frac{a^2}{2}\ln\left|\sec\theta+\tan\theta\right| + k = \frac{a^2}{2}\frac{x}{a}\frac{\sqrt{x^2-a^2}}{a} - \frac{a^2}{2}\ln\left|\frac{x}{a}+\frac{\sqrt{x^2-a^2}}{a}\right| + k =$$

$$= \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + \frac{a^2}{2}\ln|a| + k =$$

$$= \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x+\sqrt{x^2-a^2}\right| + k$$

can leave out because it is a constant

(2) $\frac{d^{71}\cos x}{dx^{71}} = \frac{d^3}{dx^3}\frac{d^{68}\cos x}{dx^{68}} = \frac{d^3\cos x}{dx^3} = \sin x$ (68 is divisible by 4)

(3) $f'(x) = A(2x\cosh x + x^2\sinh x) + B(2x\sinh x + x^2\cosh x) + C(\cosh x + x\sinh x) +$
$+ D(\sinh x + x\cosh x) + E\sinh x + F\cosh x =$

$= Ax^2\sinh x + Bx^2\cosh x + (2B+C)x\sinh x + (2A+D)x\cosh x +$
$+ (C+F)\cosh x + (D+E)\sinh x = x^2\cosh x \Rightarrow$

$A=0$, $B=1$, $C=-2$, $D=0$, $E=0$, $F=2$ $\Rightarrow$

$$\int x^2\cosh x\,dx = \int f'(x)\,dx = f(x)+k = x^2\sinh x - 2x\cosh x + 2\sinh x + k$$

(4) $P = P_0 e^{kt}$, $2014 \to t=0$, $P = 2.168e^{kt}$, $2020 \to t=6$, $2.313 = 2.168e^{6k}$,

$e^{6k} = \frac{2.313}{2.168}$, $6k = \ln\left(\frac{2.313}{2.168}\right)$, $k = \frac{1}{6}\ln\left(\frac{2.313}{2.168}\right) = 0.01079/\text{yr}$,

$2032 \to t = 18$, $P = 2.168e^{18k} = 2.633$ million

(5) $\frac{dy}{50-3y} = dx$, $u = 50-3y$, $\frac{du}{dy} = -3$, $dy = -\frac{1}{3}du$, $u_0 = 50-3(25) = -25$,

$-\frac{1}{3}\frac{du}{u} = dx$, $\frac{du}{u} = -3\,dx$, $\int_{u_0}^{u}\frac{dv}{v} = -3\int_0^x dt$, $[\ln v]_{u_0}^{u} = [\ln v]_{-25}^{50-3y} = -3[t]_0^x$,

$\ln\left(\frac{50-3y}{-25}\right) = -3x$, $\frac{3y-50}{25} = e^{-3x}$, $3y-50 = 25e^{-3x}$, $3y = 50+25e^{-3x}$,

$y = \frac{50}{3} + \frac{25}{3}e^{-3x}$

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(6) $T = T_\infty + (T_0 - T_\infty)e^{-kt}$, $T = 72 + (T_0 - 72)e^{-kt}$,

$95 = 72 + (T_0 - 72)e^{-8k}$, $23 = (T_0 - 72)e^{-8k}$
$120 = 72 + (T_0 - 72)e^{-4k}$, $48 = (T_0 - 72)e^{-4k}$ $\Big\}$ $(\div) \Rightarrow \frac{23}{48} = e^{-4k}$, $\frac{48}{23} = e^{4k}$,

$4k = \ln\left(\frac{48}{23}\right)$, $k = \frac{1}{4}\ln\left(\frac{48}{23}\right) = 0.1839$ /min, $T_0 - 72 = 23e^{8k}$,

$T_0 = 72 + 23e^{8k} = 172.17°F$

(7) $\lim_{x\to 0^+} \frac{\cos x - 1}{x^3} = \frac{0}{0} = \lim_{x\to 0^+} \frac{-\sin x}{3x^2} = \frac{0}{0} = \lim_{x\to 0^+} \frac{-\cos x}{6x} = \frac{-1}{0^+} = -\infty$

(8) $\lim_{x\to 0} \frac{\sin 6x}{\tan 3x} = \frac{0}{0} = \lim_{x\to 0} \frac{6\cos 6x}{3\sec^2 3x} = 2\lim_{x\to 0} \cos 6x \cos^2 3x = 2 \cdot 1 = 2$

(9) $\lim_{x\to 0^+} (8x)^{2x} = L$, $\lim_{x\to 0^+} \ln(8x)^{2x} = \lim_{x\to 0^+} 2x\ln(8x) = 2\lim_{x\to 0^+} x\ln(8x) = 0 \cdot -\infty =$

$= 2\lim_{x\to 0^+} \frac{\ln(8x)}{\frac{1}{x}} = \frac{-\infty}{\infty} = 2\lim_{x\to 0^+} \frac{\frac{1}{8x}\cdot 8}{-\frac{1}{x^2}} = 2\lim_{x\to 0^+} (-x) = 2 \cdot 0 = 0 = \ln L$

$\lim_{x\to 0^+} (8x)^{2x} = L = e^0 = 1$