Section 8.1 Exercises

In Exercises 1–8, the function $v(t)$ is the velocity in m/sec of a particle moving along the x-axis. Use analytic methods to do each of the following:

(a) Determine when the particle is moving to the right, to the left, and stopped.

(b) Find the particle's displacement for the given time interval. If $s(0) = 3$, what is the particle's final position?

(c) Find the total distance traveled by the particle.

1. $v(t) = 5 \cos t, \quad 0 \le t \le 2\pi$

2. $v(t) = 6 \sin 3t, \quad 0 \le t \le \pi/2$

3. $v(t) = 49 - 9.8t, \quad 0 \le t \le 10$

4. $v(t) = 6t^2 - 18t + 12, \quad 0 \le t \le 2$

5. $v(t) = 5 \sin^2 t \cos t, \quad 0 \le t \le 2\pi$

6. $v(t) = \sqrt{4 - t}, \quad 0 \le t \le 4$

7. $v(t) = e^{\sin t} \cos t, \quad 0 \le t \le 2\pi$

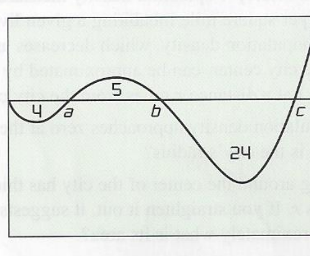
8. $v(t) = \dfrac{t}{1 + t^2}, \quad 0 \le t \le 3$

9. An automobile accelerates from rest at $1 + 3\sqrt{t}$ mph/sec for 9 seconds.
   (a) What is its velocity after 9 seconds?
   (b) How far does it travel in those 9 seconds?

10. A particle travels with velocity
$$v(t) = (t - 2) \sin t \text{ m/sec}$$
for $0 \le t \le 4$ sec.
   (a) What is the particle's displacement?
   (b) What is the total distance traveled?

11. ***Projectile*** Recall that the acceleration due to Earth's gravity is 32 ft/sec^2. From ground level, a projectile is fired straight upward with velocity 90 feet per second.
   (a) What is its velocity after 3 seconds?
   (b) When does it hit the ground?
   (c) When it hits the ground, what is the net distance it has traveled?
   (d) When it hits the ground, what is the total distance it has traveled?

In Exercises 12–16, a particle moves along the x-axis (units in cm). Its initial position at $t = 0$ sec is $x(0) = 15$. The figure shows the graph of the particle's velocity $v(t)$. The numbers are the *areas* of the enclosed regions.



12. What is the particle's displacement between $t = 0$ and $t = c$?

13. What is the total distance traveled by the particle in the same time period?

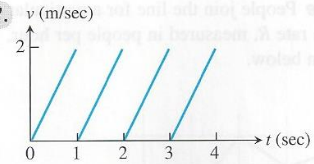
14. Give the positions of the particle at times a, b, and c.

15. Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, b]$?

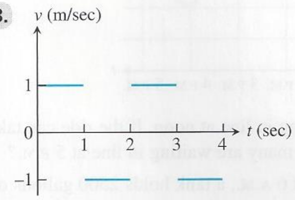
16. Approximately where does the particle achieve its greatest positive acceleration on the interval $[0, c]$?

In Exercises 17–20, the graph of the velocity of a particle moving on the x-axis is given. The particle starts at $x = 2$ when $t = 0$.
   (a) Find where the particle is at the end of the trip.
   (b) Find the total distance traveled by the particle.

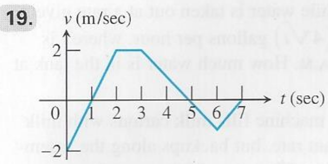
17.



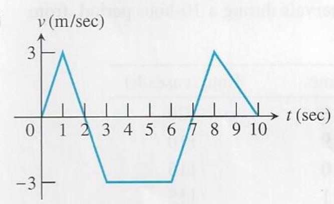
18.



19.



20.



21. ***U.S. Oil Consumption*** The rate of consumption of oil in the United States during the 1980s (in billions of barrels per year) is modeled by the function $C = 27.08 \cdot e^{t/25}$, where t is the number of years after January 1, 1980. Find the total consumption of oil in the United States from January 1, 1980, to January 1, 1990.

22. ***Home Electricity Use*** The rate at which your home consumes electricity is measured in kilowatts. If your home consumes electricity at the rate of 1 kilowatt for 1 hour, you will be charged for 1 "kilowatt-hour" of electricity. Suppose that the average consumption rate for a certain home is modeled by the function $C(t) = 3.9 - 2.4 \sin(\pi t/12)$, where $C(t)$ is measured in kilowatts and t is the number of hours past midnight. Find the average daily consumption for this home, measured in kilowatt-hours.

23. ***Light Bulbs*** A thousand compact fluorescent light bulbs from a certain company burn out at a rate of $\frac{7}{4}e^{-0.00175t}$ bulbs per day if they are left burning continually, where t is measured in days from when they are first turned on. To the nearest whole light bulb, how many will have burned out after 100 days of continuous use?

24. ***Traffic Flow*** Midday traffic through an intersection can be modeled by the function $74 + 6\cos(t/3)$ cars per minute, where t is measured in minutes after noon. Find the number of cars that pass through this intersection between noon and 12:30 P.M.

25. *Getting onto the Ride* People join the line for a particular amusement park ride at a rate R, measured in people per hour, given by the graph shown below.

There are 500 people waiting in line at noon. If the ride can take 1500 people per hour, how many are waiting in line at 5 P.M.?

26. *Water In, Water Out* At 6 A.M., a tank holds 2500 gallons of water. Over the next 18 hours, it is being filled at a constant rate of 100 gallons per hour while water is taken out at a rate given by $W(t) = 120 + 60\sin(4\sqrt{t})$ gallons per hour, where t is measured in hours after 6 A.M. How much water is in the tank at midnight, when $t = 18$?

27. *Filling Milk Cartons* A machine fills milk cartons with milk at an approximately constant rate, but backups along the assembly line cause some variation. The rates (in cases per hour) are recorded at hourly intervals during a 10-hour period, from 8:00 A.M. to 6:00 P.M.

Time	Rate (cases/h)
8	120
9	110
10	115
11	115
12	119
1	120
2	120
3	115
4	112
5	110
6	121

Use the Trapezoidal Rule with $n = 10$ to determine approximately how many cases of milk were filled by the machine over the 10-hour period.

28. *Fuel Consumption* The table shows the rate of fuel consumption in a small plane at different times during a 90-minute flight, where t is measured in minutes since the start of the flight. Use a trapezoidal approximation to estimate the total fuel consumption during the flight. (Watch your units.)

Time (minutes)	Fuel Consumption (gallons per hour)
0	4
10	16
30	12
45	6
60	7
80	5
90	4

29. *Hot Dogs* A popular hot dog stand opens at noon. At different times between noon and 3 P.M., the manager stops by to measure how fast the hot dogs are selling, with the results given in the table. Use a trapezoidal approximation to estimate the total number of hot dogs sold during the first three hours.

Time (minutes after noon)	Rate of Hot Dog Sales (hot dogs per minute)
0	3
10	6
30	18
45	16
60	12
90	8
120	10
180	6

30. Writing to Learn As a school project, Anna accompanies her mother on a trip to the grocery store and keeps a log of the car's speed at 10-second intervals. Explain how she can use the data to estimate the distance from her home to the store. What is the connection between this process and the definite integral?

31. *Finding Mass* A two-meter-long rod of uniform diameter 6 cm has density $3/(5 + x)$ grams per cubic centimeter at distance x, measured in meters, from the left-hand endpoint. Find the total mass of the rod.

32. *Weight of Air* Within the troposphere (up to about 17 km above sea level), the density of air at h meters above sea level is given by

$$\text{density of air} = 4.17 \times 10^{-11}(288.15 - 0.0065h)^{4.256}\ \text{kg/m}^3.$$

Find the mass of a cylindrical column of air one meter in diameter from sea level up to 10 km.

33. *Population Density* Population density measures the number of people per square mile inhabiting a given living area. Washerton's population density, which decreases as you move away from the city center, can be approximated by the function $10{,}000(2 - r)$ at a distance r miles from the city center.

(a) If the population density approaches zero at the edge of the city, what is the city's radius?

(b) A thin ring around the center of the city has thickness Δr and radius r. If you straighten it out, it suggests a rectangular strip. Approximately what is its area?

(c) Writing to Learn Explain why the population of the ring in part (b) is approximately

$$10{,}000(2 - r)(2\pi r)\Delta r.$$

(d) Estimate the total population of Washerton by setting up and evaluating a definite integral.

34. *Oil Flow* Oil flows through a cylindrical pipe of radius 3 inches, but friction from the pipe slows the flow toward the outer edge. The speed at which the oil flows at a distance r inches from the center is $8(10 - r^2)$ inches per second.

(a) In a plane cross section of the pipe, a thin ring with thickness Δr at a distance r inches from the center approximates a rectangular strip when you straighten it out. What is the area of the strip (and hence the approximate area of the ring)?

(b) Explain why we know that oil passes through this ring at approximately $8(10 - r^2)(2\pi r)\Delta r$ cubic inches per second.

(c) Set up and evaluate a definite integral that will give the rate (in cubic inches per second) at which oil is flowing through the pipe.

35. ***Hooke's Law*** A certain spring requires a force of 6 N to stretch it 3 cm beyond its natural length.

(a) What force would be required to stretch the spring 9 cm beyond its natural length?

(b) What would be the work done in stretching the spring 9 cm beyond its natural length?

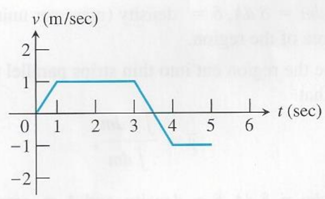
36. ***Hooke's Law*** Hooke's Law also applies to *compressing* springs; that is, it requires a force of kx to compress a spring a distance x from its natural length. Suppose a 10,000-lb force compressed a spring from its natural length of 12 inches to a length of 11 inches. How much work was done in compressing the spring

(a) the first half-inch? **(b)** the second half-inch?

Standardized Test Questions

You may use a graphing calculator to solve the following problems.

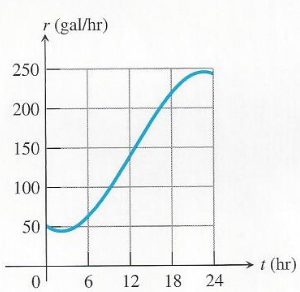
37. True or False The figure below shows the velocity for a particle moving along the x-axis. The displacement for this particle is negative. Justify your answer.



38. True or False If the velocity of a particle moving along the x-axis is always positive, then the displacement is equal to the total distance traveled. Justify your answer.

39. Multiple Choice The graph below shows the rate at which water is pumped from a storage tank. Approximate the total gallons of water pumped from the tank in 24 hours.

(A) 600 **(B)** 2400 **(C)** 3600 **(D)** 4200 **(E)** 4800



40. Multiple Choice The data for the acceleration $a(t)$ of a car from 0 to 15 seconds are given in the table below. If the velocity at $t = 0$ is 5 ft/sec, which of the following gives the approximate velocity at $t = 15$ using the Trapezoidal Rule?

(A) 47 ft/sec **(B)** 52 ft/sec **(C)** 120 ft/sec

(D) 125 ft/sec **(E)** 141 ft/sec

t (sec)	0	3	6	9	12	15
$a(t)$ (ft/sec^2)	4	8	6	9	10	10

41. Multiple Choice The rate at which customers arrive at a counter to be served is modeled by the function F defined by $F(t) = 12 + 6\cos\left(\frac{t}{\pi}\right)$ for $0 \le t \le 60$, where $F(t)$ is measured in customers per minute and t is measured in minutes. To the nearest whole number, how many customers arrive at the counter over the 60-minute period?

(A) 720 **(B)** 725 **(C)** 732

(D) 744 **(E)** 756

42. Multiple Choice Pollution is being removed from a lake at a rate modeled by the function $y = 20e^{-0.5t}$ tons/yr, where t is the number of years since 2005. Estimate the amount of pollution removed from the lake between 2005 and 2015. Round your answer to the nearest ton.

(A) 40 **(B)** 47 **(C)** 56

(D) 61 **(E)** 71

Extending the Ideas

43. ***Inflation*** Although the economy is continuously changing, we analyze it with discrete measurements. The following table records the *annual* inflation rate as measured each month for 13 consecutive months. Use the Trapezoidal Rule with $n = 12$ to find the overall inflation rate for the year.

Month	Annual Rate
January	0.04
February	0.04
March	0.05
April	0.06
May	0.05
June	0.04
July	0.04
August	0.05
September	0.04
October	0.06
November	0.06
December	0.05
January	0.05

44. ***Inflation Rate*** The table below shows the *monthly* inflation rate (in *thousandths*) for energy prices for thirteen consecutive months. Use the Trapezoidal Rule with $n = 12$ to approximate the *annual* inflation rate for the 12-month period running from the middle of the first month to the middle of the last month.

Month	Monthly Rate (in thousandths)
January	3.6
February	4.0
March	3.1
April	2.8
May	2.8
June	3.2
July	3.3
August	3.1
September	3.2
October	3.4
November	3.4
December	3.9
January	4.0

45. In Exercise 25, the rate at which people join the line from noon $(t = 0)$ until 5 P.M. $(t = 5)$ can be described by the piecewise defined function

$$R(t) = \begin{cases} 2000 - 250t, & 0 \le t < 2 \\ 500 + 500t, & 2 \le t < 3 \\ 5000 - 1000t, & 3 \le t < 4 \\ -1000 + 500t, & 4 \le t \le 5. \end{cases}$$

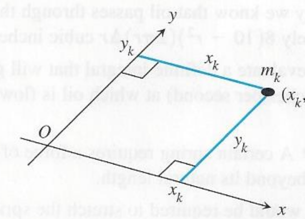
Given that 500 people are waiting in line at noon, find the piecewise defined function $L(T)$ that describes the total number of people waiting in line at time T, $0 \le T \le 5$.

46. Continuing Exercises 25 and 45:

(a) Since the ride can accommodate 1500 people per hour, find the piecewise defined function that describes the wait time in minutes for a person who arrives at time T.

(b) Find the average wait time between noon and 5 P.M.

47. ***Center of Mass*** Suppose we have a finite collection of masses in the coordinate plane, the mass m_k located at the point (x_k, y_k) as shown in the figure.



Each mass m_k has **moment $m_k y_k$ with respect to the x-axis** and **moment $m_k x_k$ about the y-axis.** The moments of the entire system with respect to the two axes are

$$\text{Moment about } x\text{-axis: } M_x = \sum m_k y_k,$$

$$\text{Moment about } y\text{-axis: } M_y = \sum m_k x_k.$$

The **center of mass** is $(\bar{x}, \bar{y})$ where

$$\bar{x} = \frac{M_y}{M} = \frac{\sum m_k x_k}{\sum m_k} \quad \text{and} \quad \bar{y} = \frac{M_x}{M} = \frac{\sum m_k y_k}{\sum m_k}.$$

Suppose we have a thin, flat plate occupying a region in the plane.

(a) Imagine the region cut into thin strips parallel to the y-axis. Show that

$$\bar{x} = \frac{\int x\,dm}{\int dm},$$

where $dm = \delta\,dA$, $\delta =$ density (mass per unit area), and $A =$ area of the region.

(b) Imagine the region cut into thin strips parallel to the x-axis. Show that

$$\bar{y} = \frac{\int y\,dm}{\int dm},$$

where $dm = \delta\,dA$, $\delta =$ density, and $A =$ area of the region.

In Exercises 48 and 49, use Exercise 47 to find the center of mass of the region with given density.

48. the region bounded by the parabola $y = x^2$ and the line $y = 4$ with constant density δ

49. the region bounded by the lines $y = x$, $y = -x$, $x = 2$ with constant density δ