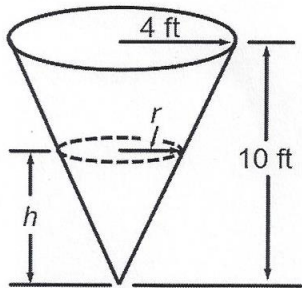


8.1. ACCUMULATIONS

AP CALCULUS AB

At time $t = 0$ min the conical tank shown is filled with water. Water is drained from the bottom of the tank at a rate of $\frac{1}{6}\sqrt{h}$ ft³/min, where $h = h(t)$ is the current depth of the water.



Find $h = h(t)$. *Hint:* Use similar triangles to express the volume of water in the tank in terms of h only. Also find the time it takes to drain the tank.

$$V = \frac{1}{3}\pi r^2 h, \quad \frac{r}{4} = \frac{h}{10}, \quad r = \frac{2}{5}h, \quad V = \frac{1}{3}\pi \cdot \frac{4}{25}h^2 \cdot h = \frac{4\pi}{75}h^3,$$

$$\frac{dV}{dt} = -\frac{1}{6}\sqrt{h} = \frac{4\pi}{75} \cdot 3h^2 \frac{dh}{dt} = \frac{4\pi}{25}h^2 \frac{dh}{dt}, \quad -\frac{1}{6}\sqrt{h} = \frac{4\pi}{25}h^2 \frac{dh}{dt},$$

$$-\frac{24\pi}{25} \frac{h^2}{\sqrt{h}} dh = dt, \quad -\frac{24\pi}{25} h^{3/2} dh = dt, \quad -\frac{24\pi}{25} \int_{10}^h z^{3/2} dz = \int_0^t dx,$$

$$-\frac{24\pi}{25} \left[\frac{2}{5} z^{5/2} \right]_{10}^h = -\frac{48\pi}{125} \left[z^{5/2} \right]_{10}^h = \left[x \right]_0^t, \quad -\frac{48\pi}{125} (h^{5/2} - 10^{5/2}) = t,$$

$$h^{5/2} - 10^{5/2} = -\frac{125t}{48\pi}, \quad h^{5/2} = 10^{5/2} - \frac{125t}{48\pi}, \quad h = \left(10^{5/2} - \frac{125t}{48\pi} \right)^{2/5}$$

Time to empty:
 $h=0$ $\frac{125t}{48\pi} = 10^{5/2}, \quad t = \frac{48\pi \cdot 10^{5/2}}{125} = 381.488 \text{ min} = 6.358 \text{ hrs}$
 $= 6:21:29$