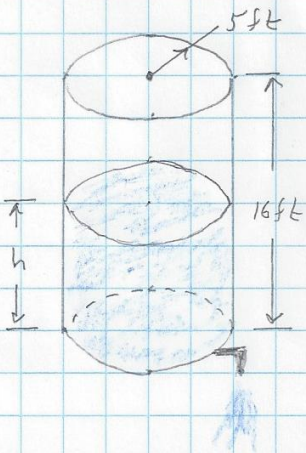


8.1. Accumulations

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Example. The drain at the bottom of the cylindrical water tank releases water at a rate of $200\sqrt{h} \frac{\text{ft}^3}{\text{hr}}$, where h is the current depth of the water. Assuming that the tank is full at $t=0$, find the depth as a function of time, i.e., $h=h(t)$, and find how long it takes for the tank to empty.



SOLUTION:

$$V = \pi r^2 h = 25\pi h, \quad \frac{dV}{dt} = 25\pi \frac{dh}{dt} = -200\sqrt{h},$$
$$-\frac{\pi}{8} \frac{dh}{dt} = \sqrt{h}, \quad \frac{dh}{\sqrt{h}} = -\frac{8}{\pi} dt, \quad \int_{16}^h \frac{dz}{\sqrt{z}} = -\frac{8}{\pi} \int_0^t dx,$$

$$\left[2\sqrt{z} \right]_{16}^h = -\frac{8}{\pi} [x]_0^t, \quad \left[\sqrt{z} \right]_{16}^h = -\frac{4}{\pi} [x]_0^t, \quad \sqrt{h} - 4 = -\frac{4}{\pi} t,$$

$$\sqrt{h} = 4 \left(1 - \frac{t}{\pi} \right), \quad h = 16 \left(1 - \frac{t}{\pi} \right)^2$$

Tank is empty when $h=0 \Rightarrow t = \pi \text{ hrs} = 3:08:30$