

AP CALCULUS AB

ACCUMULATIONS – II

1) Verify the integral

$$\int t^2 e^{-t/5} dt = \int f'(t) dt = f(t) = -5 \left(t^2 e^{-t/5} + 10t e^{-t/5} + 50 e^{-t/5} \right)$$

by differentiation.

$$\begin{aligned} f'(t) &= -5 \left[2t e^{-t/5} + t^2 \cdot -\frac{1}{5} e^{-t/5} \right. \\ &\quad \left. + 10 \left(e^{-t/5} + t \cdot -\frac{1}{5} e^{-t/5} \right) \right. \\ &\quad \left. + 50 \cdot -\frac{1}{5} e^{-t/5} \right] = \\ &= -5 \left[2t e^{-t/5} - \frac{1}{5} t^2 e^{-t/5} \right. \\ &\quad \left. + 10 e^{-t/5} - 2t e^{-t/5} - 10 e^{-t/5} \right] = \\ &= -5 \left[-\frac{1}{5} t^2 e^{-t/5} \right] = t^2 e^{-t/5} \end{aligned}$$

$P = P(t) \equiv$ number of people in the park.

$$P(6) = P(0) + \int_0^6 E(t) dt - \int_0^6 L(t) dt,$$

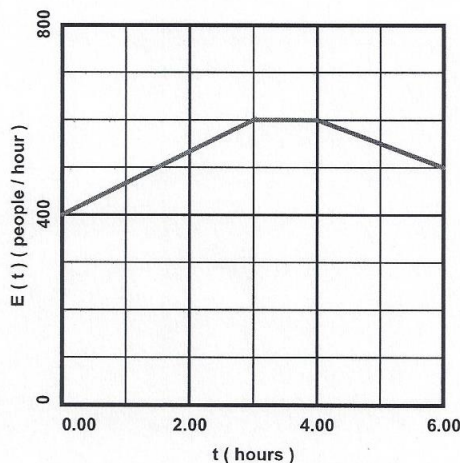
Use trapezoids and a rectangle...

$$\int_0^6 E(t) dt = \frac{1}{2} (400 + 600)(3) + (1)(600) + \frac{1}{2} (600 + 500)(2) = 1500 + 600 + 1100 = 3200$$

$$\int_0^6 L(t) dt = \left[100t - 250 \left(t^2 + 10t + 50 \right) e^{-t/5} \right]_0^6 = 2106.41 = 2106$$

$$P(6) = 3000 + 3200 - 2106 = 4094 \text{ people}$$

2) The graph of $E = E(t)$ shown



gives the number of people per hour that enter an amusement park from noon ($t = 0$) until 6:00pm ($t = 6$). During that same time period, the number of people leaving the park $L = L(t)$, in people per hour, is given by

$$L = 50 \left(2 + t^2 e^{-t/5} \right).$$

If at noon there were 3000 people in the park, how many are in the park at 6:00pm? You will need to use the integral from problem 1.