

8.1. Accumulations - II

10F1

Example: A holding pond at a mine has $E = E(t)$ gal/min of water entering it, given by $E = -\frac{1}{3}t + 40$ on $t \in [0, 60]$ min. A pump removes $R = R(t)$ gal/min as given by the table. If at $t=0$ the holding pond contains 5000 gallons, how many gallons does it contain at $t=60$ min?

t (min)	R (gal/min)
0	58
5	60
10	65
15	64
20	58
25	57
30	55
35	55
40	59
45	60
50	60
55	63
60	63

SOLUTION:

$G(t) \equiv$ gallons of water in the pond.

$$G(60) = G(0) + \int_0^{60} E(t) dt - \int_0^{60} R(t) dt.$$

$$\int_0^{60} E(t) dt = \int_0^{60} \left(-\frac{1}{3}t + 40\right) dt =$$

$$= \left[-\frac{1}{6}t^2 + 40t\right]_0^{60} = 1800$$

Estimate $\int_0^{60} R(t) dt$ with Simpson's Rule, $\Delta x = 10$, i.e., 6 equal subdivisions...

$$\int_0^{60} R(t) dt = \frac{1}{6} \left[58 + 4(60) + 2(65) + 4(64) + 2(58) + 4(57) + 2(55) + 4(55) + 2(59) + 4(60) + 2(60) + 4(63) + 63 \right] \Delta x = 3585$$

$$G(60) = 5000 + 1800 - 3585 = 3215 \text{ gal} \leftarrow$$