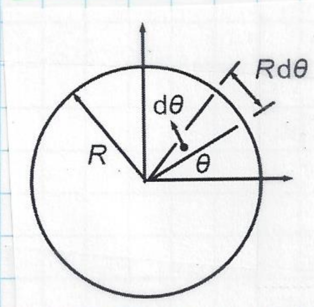
8. Circles and Spheres

Example #1. Use calculus to find the formula for the circumference of a circle.

SOLUTION:



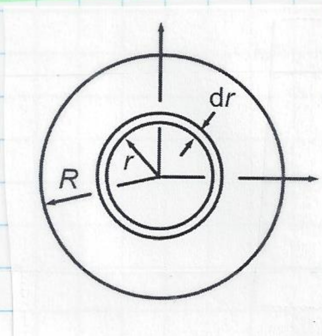
$dc \equiv$ differential of circumference

$dc = R\,d\theta \equiv$ radius · angle in radians

$$C = \int dc = R\int_0^{2\pi} d\theta = R\left[\theta\right]_0^{2\pi} = R\cdot 2\pi = 2\pi R$$

Example #2. Use calculus to find the formula for the area of a circle.

SOLUTION:



The differential of area dA is the ring shown on the figure. On the differential scale, everything is linear, so...

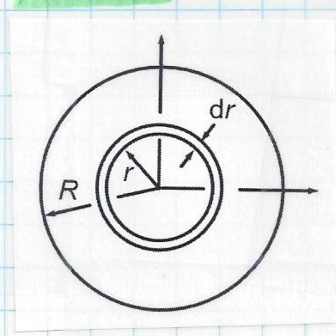
dA = length · width = circumference · dr = $2\pi r \cdot dr$

$$A = \int dA = 2\pi \int_0^R r\,dr = 2\pi\left[\frac{r^2}{2}\right]_0^R = \pi R^2$$

From the worksheet, the surface area of a sphere of radius r is $S = 4\pi r^2$.

Example #3. Use calculus to find the formula for the volume of a sphere.

SOLUTION:



The differential of volume dV is the spherical shell shown on the figure. On the differential scale, everything is linear (i.e., flat), so...

dV = base area · height = surface area · dr = $4\pi r^2 \cdot dr$

$$V = \int dV = 4\pi\int_0^R r^2\,dr = 4\pi\left[\frac{r^3}{3}\right]_0^R = 4\pi\cdot\frac{R^3}{3} = \frac{4}{3}\pi R^3$$