AP CALCULUS AB

CIRCLES AND SPHERES

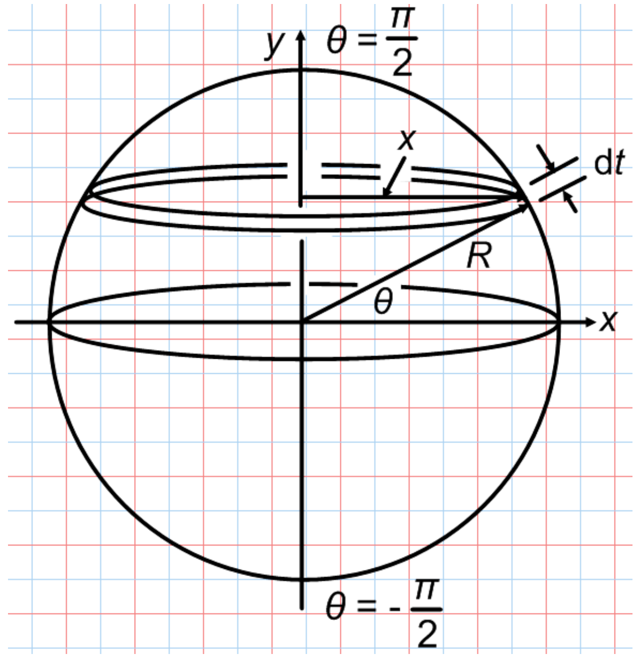
1) Verify the integral

$$\int \cos^3\theta \ \mathrm{d}\theta = \sin\theta - \frac{1}{3}\sin^3\theta$$

by differentiation.

Hint: use $\sin^2\theta + \cos^2\theta = 1$.

Surface Area of a Sphere:

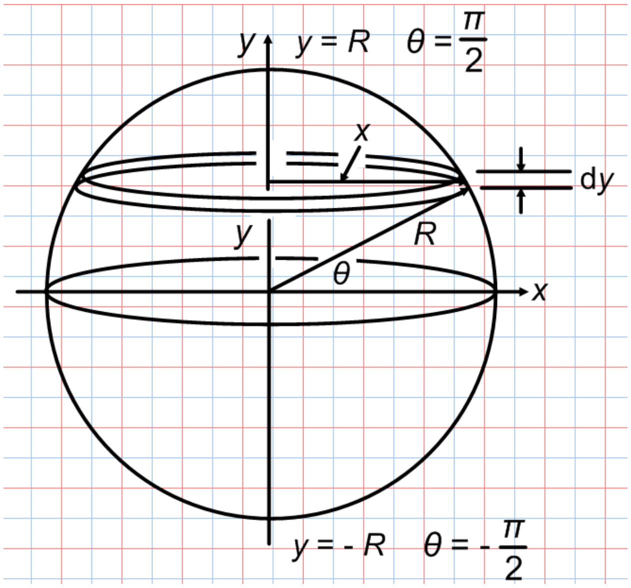


The differential of surface area dS is the ring shown on the surface of the sphere. On the differential scale, everything is linear, *i.e.*, flat. Accordingly

$$\mathrm{d}S = \text{length} \cdot \text{width} = \text{circumference} \cdot \mathrm{d}t =$$
$$= 2\pi x \cdot R\mathrm{d}\theta \ .$$

2) Use $x = R\cos\theta$ to set up an integral for $S = \int \mathrm{d}S$ in terms of R and θ, and then integrate from $\theta = -{}^{\pi}/_{2}$ to $\theta = {}^{\pi}/_{2}$ to find the well-known formula for the surface area of a sphere.

Volume of a Sphere:



The differential of volume $\mathrm{d}V$ is the cylinder in the sphere as shown on the figure. So,

$\mathrm{d}V$ = base area · height = $\pi x^2 \cdot \mathrm{d}y$.

3) Use $x = R\cos\theta$ and $y = R\sin\theta$ to set up an integral for $V = \int \mathrm{d}V$ in terms of R and θ, and then integrate from $\theta = -\pi/2$ to $\theta = \pi/2$ to find the well-known formula for the volume of a sphere.

Hint: you will need to use the integral from problem **1**.