AP CALCULUS AB

HOMEWORK #13

8.1. Accumulations

Supplemental Problems
1

8.1. Accumulations – II

Section 8.1 Exercises, pg. 392
$12^{1,2} - 16^{1}$, 21, 24, 27

Supplemental Problems
2

8.2. Areas Between Curves

Section 8.2 Exercises, pg. 402
2^{3}, 3, 9^{4}, 10^{5}, 19, 39

Supplemental Problems
3

8. Circles and Spheres

Supplemental Problems
4

Notes:

1. Recall that

$$v = \frac{\mathrm{d}x}{\mathrm{d}t} \text{ and } a = \frac{\mathrm{d}v}{\mathrm{d}t}.$$

2. For problem 12, find the position at $t = c$, not the displacement.

3. An easy way to integrate $\sin^2 t$ is to use the identity

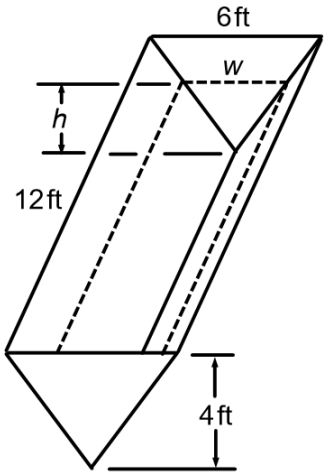
$$\sin^2 t = \frac{1}{2}(1 - \cos 2t).$$

4. Do the problem two ways, *viz.*, by integrating with respect to x, and by integrating with respect to y.

5. Integrate with respect to y.

Supplemental Problems:

1) A water trough in the shape of a triangular prism is filled with water at time $t = 0$. Water is being drained from the trough at a rate of $3\sqrt{h}$ $\mathrm{ft}^3/\mathrm{min}$, where h is the current height of the water.



a) Find $h = h(t)$. *Hint:* express the volume V of the water in terms of h only by using similar triangles.

b) How long does it take for the trough to drain completely?

2) A water storage tank is empty at time $t = 0$. During $t \in [0, 12]$ minutes, the tank is being filled at a rate of F gal/min, where

$$F(t) = -\frac{5}{3}t + 60.$$

After $t = 12$, it is stopped being filled. At time $t = 6$ minutes, a pump is started, which removes water at a rate of P gal/min, where

$$P(t) = 5t + 20.$$

Let $V = V(t)$ denote the volume of water in the tank in gallons, with t being in minutes.

a) Write $V'(t)$ as a piecewise-defined function.

b) Calculate $V = V(t)$.

c) At what time is the tank completely drained?

d) At what time is the water in the tank maximum, and what is the maximum volume? Justify your answer with values of V, V' and V''.

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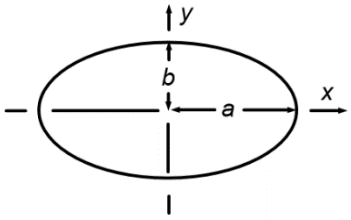
3) The equation of an ellipse is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\,.$$



a) Solve the equation of the ellipse for y.

b) Find the formula for the area of the ellipse by integration. Recall that

$$\int \sqrt{a^2-x^2}\,\mathrm{d}x =$$
$$=\frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)+\frac{x}{2}\sqrt{a^2-x^2}\,.$$

4) Using the cylindrical disk as the differential of volume dV as pictured, integrate with respect to y to obtain the formula for the volume of a sphere of radius R. *Hint:* the bounding circle of the sphere in the xy – plane is $x^2+y^2=R^2$.