Supplemental

8.1. Accumulations

(1) $V = (\text{base area})\cdot\text{height} = (\frac{1}{2}wh)(12) = 6wh.$ $\frac{w}{6} = \frac{h}{4}$, $w = \frac{3}{2}h$,

(a)

$V = 6wh = 6\left(\frac{3}{2}h\right)h = 9h^2$, $\frac{dV}{dt} = 9\cdot 2h\frac{dh}{dt} = -3\sqrt{h}$, $h\frac{dh}{dt} = -\frac{1}{6}\sqrt{h}$,

$\sqrt{h}\,dh = -\frac{1}{6}dt$, $\int_4^h \sqrt{z}\,dz = -\frac{1}{6}\int_0^t dx$, $\left[\frac{2}{3}z^{3/2}\right]_4^h = -\frac{1}{6}\left[x\right]_0^t$,

$\left[z^{3/2}\right]_4^h = -\frac{1}{4}\left[x\right]_0^t$, $h^{3/2} - 8 = -\frac{1}{4}t$, $h^{3/2} = 8 - \frac{1}{4}t$, $h = \left(8 - \frac{1}{4}t\right)^{2/3}$ ◄

(b) $h = 0 \Rightarrow 8 - \frac{1}{4}t = 0$, $t = 32$ min ◄

pg. 392 8.1. Accumulations – II

$v = \frac{dx}{dt}$

(12) $dx = v\,dt$, $x(t) = x(0) + \int_0^t v\,d\tau = 15 + \int_0^c v\,d\tau = 15 - 4 + 5 - 24 = -8$ cm $= x(c)$ ◄

(13) $4 + 5 + 24 = 33$ cm ◄

(14) $x(a) = x(0) + \int_0^a v\,dt = 15 - 4 = 11$ cm ◄ $x(b) = x(0) + \int_0^b v\,dt = 15 - 4 + 5 = 16$ cm ◄

(15) $a = \frac{dv}{dt}$, for $t \in [0, b]$ largest slope is at $t = a$ ◄

(16) largest slope is at $t = c$ ◄

(21) $1980 \rightarrow t = 0$, $1990 \rightarrow t = 10$, consumption $= 27.08\int_0^{10} e^{t/25} = 27.08\left[25e^{t/25}\right]_0^{10} =$

$= 332.97$ billion barrels ◄

(24) $c \equiv$ # of cars, noon $\rightarrow t = 0$ min, 12:30 pm $\rightarrow t = 30$ min,

$c = \int_0^{30}\left(74 + 6\cos\frac{t}{3}\right)dt = \left[74t + 18\sin\frac{t}{3}\right]_0^{30} = 2210$ cars ◄

(27) $\Delta x = 1$, cases $= \frac{1}{2}[120 + 2(110) + 2(115) + 2(115) + 2(119) + 2(120) + 2(120)$
$+ 2(115) + 2(112) + 2(110) + 121]\Delta x = 1156.5 \rightarrow 1156$ cases ◄

Supplemental

(2)

(a)

$$V'(t) = \begin{cases} F(t), & 0 \le t \le 6 \\ F(t) - p(t), & 6 < t \le 12 \\ -p(t), & 12 < t \end{cases} = \begin{cases} -\frac{5}{3}t + 60, & 0 \le t \le 6 \\ -\frac{20}{3}t + 40, & 6 < t \le 12 \\ -5t - 20, & 12 < t \end{cases}$$

(b) $0 \le t \le 6$: $V(t) = \int_0^t \left(-\frac{5}{3}\tau + 60\right) d\tau = \left[-\frac{5}{6}\tau^2 + 60\tau\right]_0^t = -\frac{5}{6}t^2 + 60t$

$V(6) = 330$, $6 < t \le 12$: $V(t) = V(6) + \int_6^t \left(-\frac{20}{3}\tau + 40\right) d\tau = 330 + \left[-\frac{10}{3}\tau^2 + 40\tau\right]_6^t =$

$= 330 - \frac{10}{3}t^2 + 40t - 120 = -\frac{10}{3}t^2 + 40t + 210$, $V(12) = 210$, $12 < t$:

$V(t) = V(12) + \int_{12}^t (-5\tau - 20)\, d\tau = 210 + \left[-\frac{5}{2}\tau^2 - 20\tau\right]_{12}^t = 210 - \frac{5}{2}t^2 - 20t - {}^{-}600 =$

$= -\frac{5}{2}t^2 - 20t + 810$

$$V(t) = \begin{cases} -\frac{5}{6}t^2 + 60t, & 0 \le t \le 6 \\ -\frac{10}{3}t^2 + 40t + 210, & 6 < t \le 12 \\ -\frac{5}{2}t^2 - 20t + 810, & 12 < t \end{cases} \blacktriangleleft$$

(c) $-\frac{5}{2}t^2 - 20t + 810 = 0$, $t = \dfrac{20 \pm \sqrt{20^2 - 4(-\frac{5}{2})(810)}}{2(-\frac{5}{2})} = \dfrac{20 \pm \sqrt{8500}}{-5}$ $t = -22.439$ $t = 14.439$

$t = 14.439$ min $= 14:26$ $\blacktriangleleft$

(d) Interior points: $-\frac{5}{3}t + 60 = V'(t) = 0$, $t = 36$ out of range for $t \in [0,6]$.

$-\frac{20}{3}t + 40 = V'(t) = 0$, $t = 6$, $V''(6^+) = -\frac{20}{3}$

$-5t - 20 = V'(t) = 0$, $t = -4$ out of range for $12 < t$

Kinks: $V'(6^-) = 50$, $V'(6^+) = 0$ $\quad$ $t = 6$ is max $\Rightarrow V_{max} = V(6) = 330$ gal $\blacktriangleleft$

$V'(12^-) = -40$, $V'(12^+) = -80$ $\quad$ not an extremum

pg. 402 8.2. Areas Between Curves

(2) $\int \sin^2 t\, dt = \int \left[\frac{1}{2} - \frac{1}{2}\cos 2t\right] dt = \frac{1}{2}t - \frac{1}{4}\sin 2t$, $A = 2\int_0^{\pi/3} \left[\frac{1}{2}\sec^2 t + 4\sin^2 t\right] dt =$

$= \int_0^{\pi/3} \left[\sec^2 t + 8\sin^2 t\right] dt = \left[\tan t + 4t - 2\sin 2t\right]_0^{\pi/3} = \frac{4\pi}{3}$ $\blacktriangleleft$

(3) $A = \int_0^1 \left[y^2 - y^3\right] dy = \left[\frac{y^3}{3} - \frac{y^4}{4}\right]_0^1 = \frac{1}{12}$ $\blacktriangleleft$

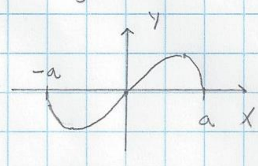
(9) with respect to x: $A = \int_0^1 \left[x - \frac{x^2}{4}\right] dx + \int_1^2 \left[1 - \frac{x^2}{4}\right] dx =$

$= \left[\frac{x^2}{2} - \frac{x^3}{12}\right]_0^1 + \left[x - \frac{x^3}{12}\right]_1^2 = \left[\frac{5}{12}\right] + \left[\frac{5}{12}\right] = \frac{5}{6}$ $\blacktriangleleft$

with respect to y...

$$A = \int_0^1 \left[2\sqrt{y} - y \right] dy = \left[\frac{4}{3}\sqrt{y^3} - \frac{y^2}{2} \right]_0^1 = \frac{5}{6}$$

(10) $$A = \int_0^1 \left[2 - y - \sqrt{y} \right] dy = \left[2y - \frac{y^2}{2} - \frac{2}{3}\sqrt{y^3} \right]_0^1 = \frac{5}{6}$$

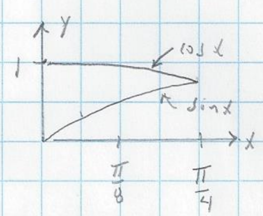
(19) 

$$A = 2\int_0^a x\sqrt{a^2 - x^2}\, dx = \cancel{2}\int_{a^2}^{0} \cancel{x} \cdot \sqrt{u} \cdot -\frac{du}{\cancel{2x}} =$$

$u = a^2 - x^2$, $\frac{du}{dx} = -2x$, $dx = -\frac{du}{2x}$, $x = 0 \to u = a^2$

$x = a \to u = 0$

$$= \int_0^{a^2} \sqrt{u}\, du = \left[\frac{2}{3}\sqrt{u^3} \right]_0^{a^2} = \frac{2}{3}a^3$$

(39) 

$$A = \int_0^{\pi/4} \left[\cos x - \sin x \right] dx = \left[\sin x + \cos x \right]_0^{\pi/4} = \sqrt{2} - 1$$

Supplemental

(3)
(a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2} = \frac{a^2}{a^2} - \frac{x^2}{a^2} = \frac{1}{a^2}(a^2 - x^2)$, $y^2 = \frac{b^2}{a^2}(a^2 - x^2)$,

$y = \frac{b}{a}\sqrt{a^2 - x^2}$, (b) $A = 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\, dx = \left[2ab\sin^{-1}\left(\frac{x}{a}\right) + \frac{2bx}{a}\sqrt{a^2 - x^2} \right]_0^a =$

$= \pi ab$

B. Circles and Spheres

Supplemental

(4) $dV = \pi x^2 dy$, $x^2 = R^2 - y^2$, $dV = \pi(R^2 - y^2)dy$, $V = \int dV = \pi\int_{-R}^{R}(R^2 - y^2)dy =$

$$= \pi\left[R^2 y - \frac{y^3}{3} \right]_{-R}^{R} = \pi\left[\frac{4}{3}R^3 \right] = \frac{4\pi}{3}R^3$$