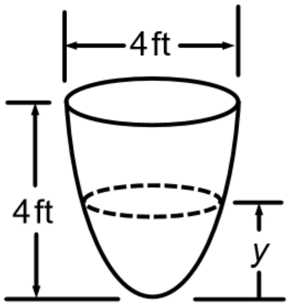
1) The paraboloid of revolution shown (which is the volume generated by rotating the curve $y = x^2$ about the y – axis), is filled with water to a depth y.



At time $t = 0$, the paraboloid is full of water, *i.e.*, $y = 4$. The volume of water in the paraboloid is

$$V = \frac{\pi}{2} y^2 .$$

Water is being drained from the volume at a rate of

$$\frac{1}{2}\sqrt{y} \ \text{ft}^3/\text{min} .$$

a) Solve for $y = y(t)$.

b) How long does it take for the paraboloid to drain completely?

2) A water storage tank is empty at time $t = 0$, and it is being filled with water at a rate F given by

$$F(t) = -\frac{1}{3}t + 50 \ \text{gal/min} ,$$

where t is in minutes. At time $t = 60$, a pump is turned on, which removes water from the tank at a rate P given by

$$P(t) = 3t - 130 \ \text{gal/min} .$$

Let $V = V(t)$ denote the volume of water in the tank, in gallons, as a function of time.

a) Write $V' = V'(t)$ as a piecewise-defined function.

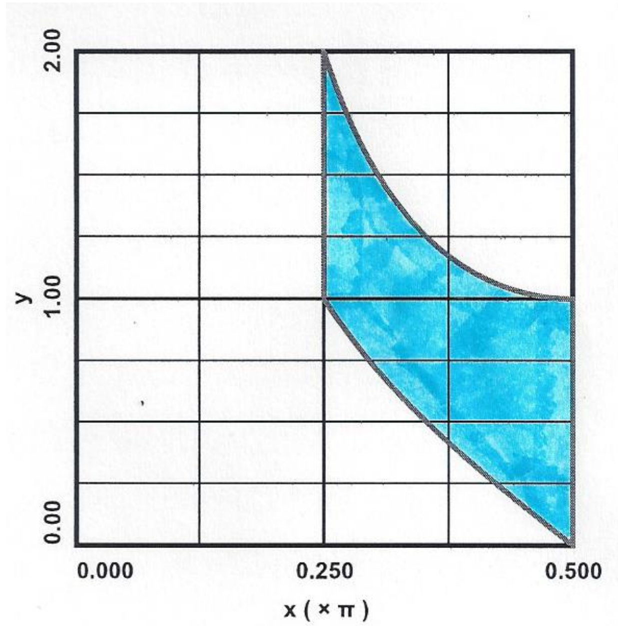
b) Calculate $V = V(t)$.

c) At what time is the amount of water in the tank maximum? Also, what is the maximum number of gallons? Justify your answers in terms of values of V and V'.

d) At what time does the tank become empty?

3) The shaded area shown is bounded by the curves $y = \cot x$, $x = \pi/2$, $y = \csc^2 x$ and $x = \pi/4$. Calculate the region's area. Integrate with respect to x. Recall that

$$\int \cot x \ \mathrm{d}x = \ln|\sin x| .$$



4) The shaded region shown is bounded by the curves $y = x^2$, $x = 2$, $y = 2x + 4$ and $x = 0$. Calculate the region's area. Integrate with respect to y.