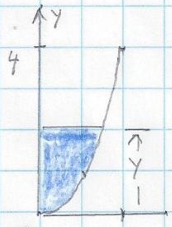
Quiz #13 Study Guide

(1)



$V = \frac{\pi}{2}y^2$, $\frac{dV}{dt} = \frac{\pi}{\cancel{2}}\cdot 2y\frac{dy}{dt} = -\frac{1}{2}\sqrt{y}$, $\pi y\,dy = -\frac{1}{2}\sqrt{y}\,dt$,

$\sqrt{y}\,dy = -\frac{dt}{2\pi}$, $\int_4^y \sqrt{z}\,dz = -\frac{1}{2\pi}\int_0^t dx$, $\left[\frac{2}{3}z^{3/2}\right]_4^y = -\frac{1}{2\pi}\left[x\right]_0^t$,

$\left[z^{3/2}\right]_4^y = -\frac{3}{4\pi}\left[x\right]_0^t$, $y^{3/2} - 8 = -\frac{3}{4\pi}t$, $y^{3/2} = 8 - \frac{3}{4\pi}t$,

$y = \left[8 - \frac{3}{4\pi}t\right]^{2/3}$ $y = 0 \Rightarrow t = \frac{32\pi}{3} = 33.51\text{ min} = 33:31$

(2) $F = -\frac{1}{3}t + 50$, $P = 3t - 130$ $(t \geq 60)$

(a)

$$V'(t) = \begin{cases} F, & 0 \leq t \leq 60 \\ F - P, & 60 < t \end{cases} = \begin{cases} -\frac{1}{3}t + 50, & 0 \leq t \leq 60 \\ -\frac{10}{3}t + 180, & 60 < t \end{cases}$$

(b) $0 \leq t \leq 60$: $V(t) = \int_0^t \left(-\frac{1}{3}z + 50\right)dz = \left[-\frac{1}{6}z^2 + 50z\right]_0^t = -\frac{1}{6}t^2 + 50t$, $V(60) = 2400$

$60 < t$: $V(t) = V(60) + \int_{60}^t \left(-\frac{10}{3}z + 180\right)dz = 2400 + \left[-\frac{5}{3}z^2 + 180z\right]_{60}^t =$

$= 2400 - \frac{5}{3}t^2 + 180t - 4800 = -\frac{5}{3}t^2 + 180t - 2400$

$$V(t) = \begin{cases} -\frac{1}{6}t^2 + 50t, & 0 \leq t \leq 60 \\ -\frac{5}{3}t^2 + 180t - 2400, & 60 < t \end{cases}$$

(c) Time when empty: $-\frac{5}{3}t^2 + 180t - 2400 = 0$, $t = \frac{-180 \pm \sqrt{180^2 - 4\left(-\frac{5}{3}\right)(-2400)}}{2\left(-\frac{5}{3}\right)} =$

$= \frac{540 \pm 3\sqrt{16,400}}{10}$ $t = 15.581$ min X
$t = 92.419$ min $= 1:32:25$

(d) $0 \leq t \leq 60$: $V' = 0 \Rightarrow t = 150$ (out of range)
$60 < t$: $V' = 0 \Rightarrow t = 54$ (out of range)

Kink: $V'(60^-) = 30$
$V'(60^+) = -20$
so $t = 60$ min is max
$V(60) = 2400$ gal is max

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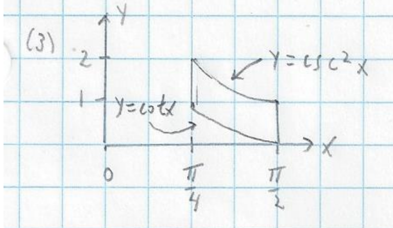
(3)

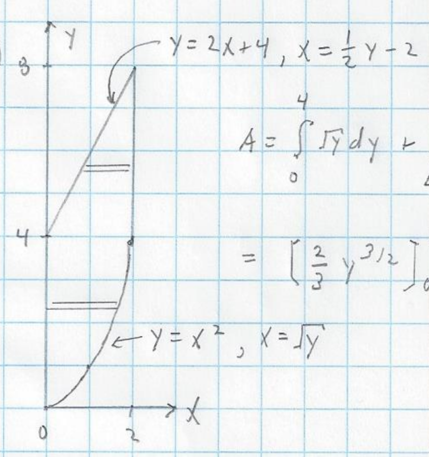


$\int \cot x \, dx = \ln|\sin x|$, $dA = (\csc^2 x - \cot x)\, dx$

$$A = \int dA = \int_{\pi/4}^{\pi/2} (\csc^2 x - \cot x)\, dx = \Big[-\cot x - \ln|\sin x| \Big]_{\pi/4}^{\pi/2} =$$

$$= 1 - \ln\sqrt{2} = 0.653\ 426\ 4097$$

(4)



$$A = \int_0^4 \sqrt{y}\, dy + \int_4^8 \left[2 - \left(\tfrac{1}{2}y - 2\right) \right] dy = \int_0^4 \sqrt{y}\, dy + \int_4^8 \left(-\tfrac{1}{2}y + 4\right) dy =$$

$$= \left[\tfrac{2}{3} y^{3/2} \right]_0^4 + \left[-\tfrac{1}{4}y^2 + 4y \right]_4^8 = \left[\tfrac{16}{3} \right] + \left[4 \right] = 9\tfrac{1}{3}$$