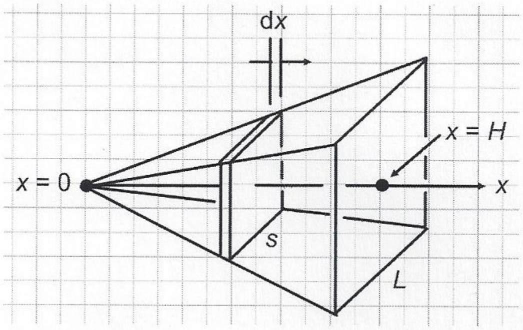
Square-Based Pyramid:



The differential of volume dV is the right, square prism shown in the figure. So,

dV = base area · height = $s^2 \cdot$ dx .

1) Find the linear relation $s = s(x)$ to express the differential dx in terms of ds, and then set up the integral $V = \int$ dV in terms of ds. Finally, find the formula for V by integrating from $s = 0$ to $s = L$.

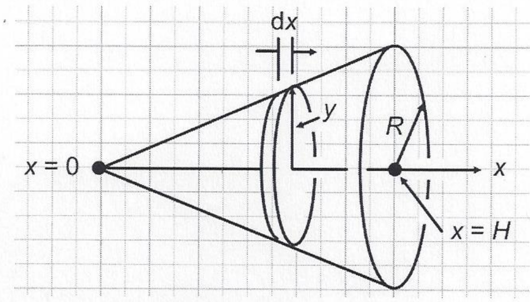
$s = mx = \frac{\Delta s}{\Delta x} x = \frac{L}{H} x$, $ds = \frac{L}{H} dx$,

$dx = \frac{H}{L} ds$, $dV = s^2 \cdot \frac{H}{L} ds = \frac{H}{L} s^2 ds$,

$V = \int dV = \frac{H}{L} \int_0^L s^2 ds = \frac{H}{L} \left[\frac{s^3}{3} \right]_0^L =$

$= \frac{HL^3}{3L} = \frac{1}{3} L^2 H$

Cone:



The differential of volume dV is the cylinder shown in the figure. So,

dV = base area · height = $\pi y^2 \cdot$ dx .

2) Find the linear relation $y = y(x)$ to express the differential dx in terms of dy, and then set up the integral $V = \int$ dV in terms of dy. Finally, find the formula for V by integrating from $y = 0$ to $y = \cancel{H}$. R

$y = mx = \frac{\Delta y}{\Delta x} x = \frac{R}{H} x$, $dy = \frac{R}{H} dx$, $dx = \frac{H}{R} dy$,

$dV = \pi y^2 \cdot \frac{H}{R} dy = \frac{\pi H}{R} y^2 dy$

$V = \int dV = \pi \frac{H}{R} \int_0^R y^2 dy = \frac{\pi H}{R} \left[\frac{y^3}{3} \right]_0^R =$

$= \frac{\pi H}{R} \cdot \frac{R^3}{3} = \frac{1}{3} \pi R^2 H$

Note! Both problems give $V = \frac{1}{3}$ (base area) · (height)