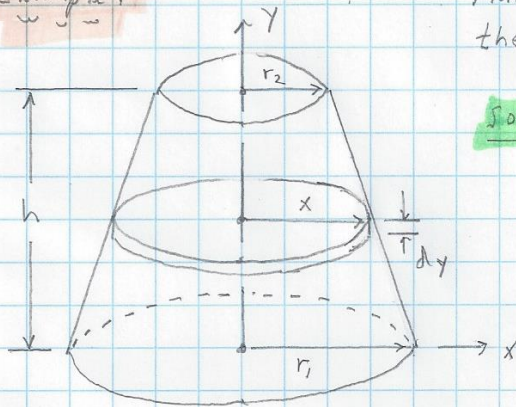


8. Pyramids, Cones and Frusta

10F1

Example.



Find the formula for the volume of the right circular frustum shown.

Solution:

The differential of volume dV is the thin cylinder shown on the figure.

$$dV = \text{base area} \cdot \text{height} = \pi x^2 dy$$

$$x = my + b, \quad m = \frac{\Delta x}{\Delta y} = \frac{r_2 - r_1}{h}$$

$$x = \left(\frac{r_2 - r_1}{h} \right) y + b, \quad r_1 = \left(\frac{r_2 - r_1}{h} \right) \cdot 0 + b, \quad b = r_1, \quad x = \left(\frac{r_2 - r_1}{h} \right) y + r_1$$

$$\frac{dx}{dy} = \frac{r_2 - r_1}{h}, \quad dy = \frac{h}{r_2 - r_1} dx, \quad dV = \pi x^2 \cdot \frac{h}{r_2 - r_1} dx$$

$$V = \int dV = \frac{\pi h}{r_2 - r_1} \int_{r_1}^{r_2} x^2 dx = \frac{\pi h}{r_2 - r_1} \left[\frac{x^3}{3} \right]_{r_1}^{r_2} = \frac{\pi h (r_2^3 - r_1^3)}{3(r_2 - r_1)}$$