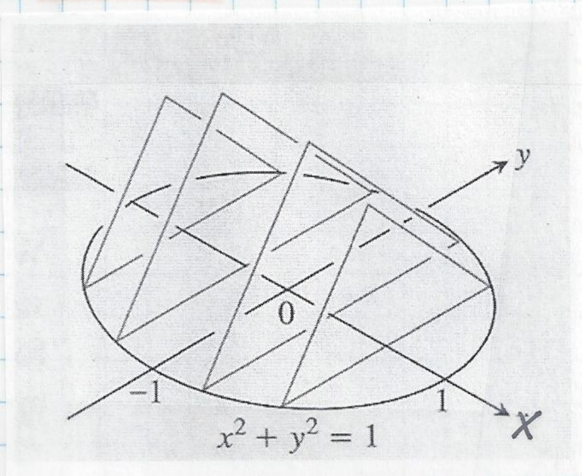
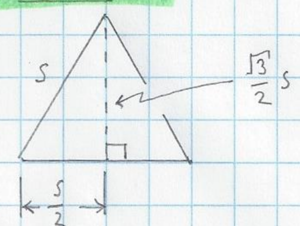
Example #1. Find the volume of the solid, whose cross-sections perpendicular to the x-axis, are equilateral triangles, with their bases running from one side of the circle to the other, as pictured.



SOLUTION:



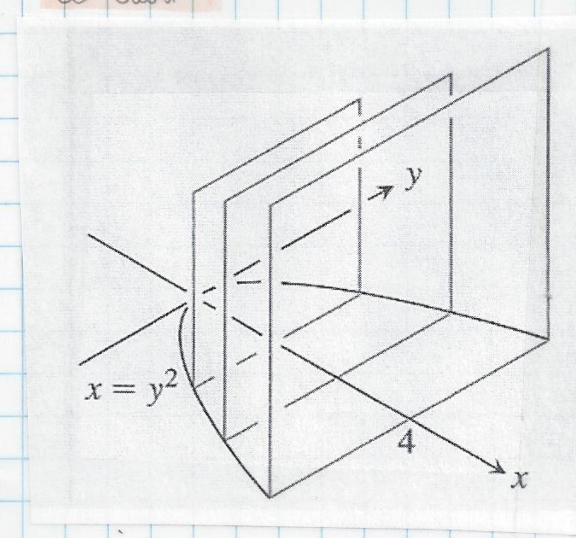
$$\text{area} = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}s \cdot \frac{\sqrt{3}}{2}s = \frac{\sqrt{3}}{4}s^2$$

$dV = (\text{area})\,dx = \frac{\sqrt{3}}{4}s^2\,dx$, $y^2 = 1 - x^2$, $\frac{1}{2}s = y$, $\frac{1}{4}s^2 = y^2 = 1 - x^2$,

$s^2 = 4(1-x^2)$, $dV = \frac{\sqrt{3}}{\cancel{4}} \cdot \cancel{4}(1-x^2)\,dx = \sqrt{3}(1-x^2)\,dx$, $V = \int dV =$

$$= \sqrt{3}\int_{-1}^{1}(1-x^2)\,dx = \sqrt{3}\left[x - \frac{x^3}{3}\right]_{-1}^{1} = \frac{4\sqrt{3}}{3}$$

Example #2. Find the volume of the solid, whose cross-sections perpendicular to the x-axis, are squares, with their sides running from one branch of the parabola to the other branch, as pictured.



SOLUTION:

$dV = s^2\,dx$, $\frac{1}{2}s = y$, $s = 2y$,

$s^2 = 4y^2 = 4x$, $dV = 4x\,dx$

$$V = \int dV = 4\int_0^4 x\,dx = 4\left[\frac{x^2}{2}\right]_0^4 = 32$$