AP CALCULUS AB

SOLIDS OF REVOLUTION

1) Verify the integral

$$\int \sin^2\left(\frac{\pi x}{8}\right) dx = \int f'(x)\,dx = f(x)$$

$$= \frac{1}{2}\left[x - \frac{8}{\pi}\sin\left(\frac{\pi x}{8}\right)\cos\left(\frac{\pi x}{8}\right)\right]$$

by differentiation. *Hint:* use

$$\sin^2\left(\frac{\pi x}{8}\right) + \cos^2\left(\frac{\pi x}{8}\right) = 1.$$

$$f'(x) = \frac{1}{2}\left[1 - \frac{8}{\pi}\left(\cos\frac{\pi x}{8}\cdot\frac{\pi}{8}\cos\frac{\pi x}{2} + \sin\frac{\pi x}{8}\cdot 0 - \sin\frac{\pi x}{8}\cdot\frac{\pi}{8}\right)\right]$$

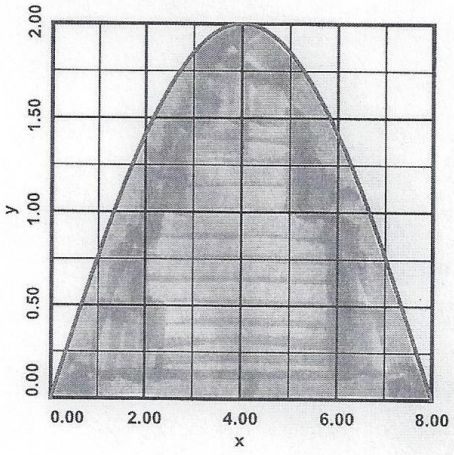
$$= \frac{1}{2}\left[1 - \cos^2\left(\frac{\pi x}{8}\right) + \sin^2\left(\frac{\pi x}{8}\right)\right] =$$

$$= \frac{1}{2}\left[\sin^2\left(\frac{\pi x}{8}\right) + \sin^2\left(\frac{\pi x}{8}\right)\right] = \sin^2\left(\frac{\pi x}{8}\right)$$

2) Find the volume of the solid generated by revolving the shaded area under

$$y = 2\sin\left(\frac{\pi x}{8}\right)$$

around the x – axis. You will need to use the integral from problem **1**.



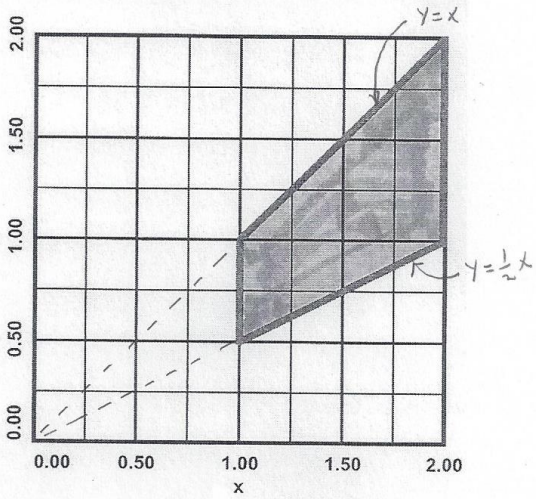
$$dV = \pi r^2 dx = \pi \cdot 4\sin^2\left(\frac{\pi x}{8}\right) dx$$

$$V = \int dV = 4\pi \int_0^8 \sin^2\left(\frac{\pi x}{8}\right) dx =$$

$$= 4\pi \cdot \frac{1}{2}\left[x - \frac{8}{\pi}\sin\left(\frac{\pi x}{8}\right)\cos\left(\frac{\pi x}{8}\right)\right]_0^8 =$$

$$= 2\pi\left[8\right] = 16\pi$$

3) Find the volume of the solid generated by revolving the shaded area around the x – axis.



$$dV = (\pi r_1^2 - \pi r_2^2)\,dx = \pi(r_1^2 - r_2^2)\,dx =$$

$$= \pi\left(x^2 - \frac{1}{4}x^2\right)dx = \frac{3\pi}{4}x^2\,dx$$

$$V = \int dV = \frac{3\pi}{4}\int_1^2 x^2\,dx = \frac{3\pi}{4}\left[\frac{x^3}{3}\right]_1^2 = \frac{\pi}{4}\cdot 7 = \frac{7\pi}{4}$$