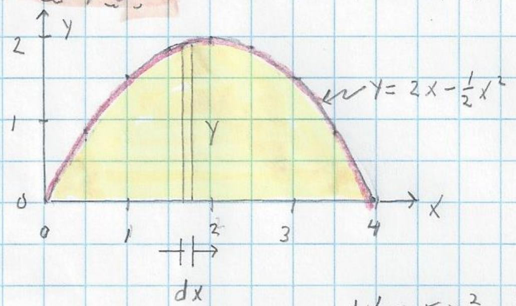
Example #1. Find the volume of the solid generated by revolving the shaded area around the x-axis.

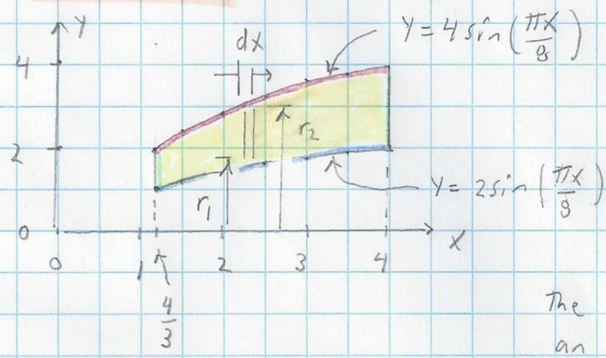


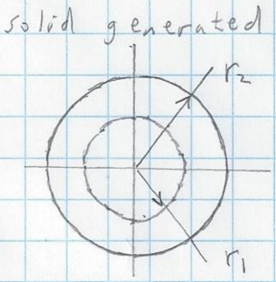
SOLUTION! The differential of volume dV is a cylinder of radius y and height dx ⇒

$$dV = \pi y^2 \cdot dx = \pi\left(2x - \tfrac{1}{2}x^2\right)^2 dx = \pi\left(4x^2 - 2x^3 + \tfrac{1}{4}x^4\right)dx$$

$$V = \int dV = \pi \int_0^4 \left(4x^2 - 2x^3 + \tfrac{1}{4}x^4\right)dx = \pi\left[\tfrac{4}{3}x^3 - \tfrac{1}{2}x^4 + \tfrac{1}{20}x^5\right]_0^4 = \frac{128}{15}\pi$$

Example #2. Find the volume of the solid generated by revolving the shaded area about the x-axis.





SOLUTION!

The differential of volume dV is an annular disk of height dx ⇒

$$dV = \left(\pi r_2^2 - \pi r_1^2\right)dx = \pi\left(r_2^2 - r_1^2\right)dx = \pi\left[16\sin^2\left(\frac{\pi x}{8}\right) - 4\sin^2\left(\frac{\pi x}{8}\right)\right]dx =$$

$$= 12\pi\sin^2\left(\frac{\pi x}{8}\right)dx$$

From the worksheet $\int \sin^2\left(\frac{\pi x}{8}\right)dx = \frac{1}{2}\left[x - \frac{8}{\pi}\sin\left(\frac{\pi x}{8}\right)\cos\left(\frac{\pi x}{8}\right)\right]$

$$V = \int dV = 12\pi\int_{4/3}^{4}\sin^2\left(\frac{\pi x}{8}\right)dx = 6\pi\left[x - \frac{8}{\pi}\sin\left(\frac{\pi x}{8}\right)\cos\left(\frac{\pi x}{8}\right)\right]_{4/3}^{4} =$$

$$= \frac{48}{3}\pi + 12\sqrt{3} = 71.050\,092\,15$$