AP CALCULUS AB

CYLINDRICAL SHELLS

1) Verify the integral

$$\int x \sin\left(\frac{\pi x}{4}\right) dx = \int f'(x)\,dx = f(x) =$$

$$= -\frac{4}{\pi} x \cos\left(\frac{\pi x}{4}\right) + \frac{16}{\pi^2} \sin\left(\frac{\pi x}{4}\right)$$

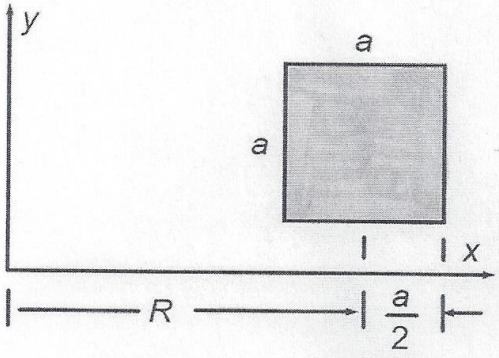
by differentiation.

$$f'(x) = -\frac{4}{\pi}\left[\cos\left(\frac{\pi x}{4}\right) + x \cdot -\sin\left(\frac{\pi x}{4}\right) \cdot \frac{\pi}{4}\right] + \frac{16}{\pi^2}\left[\cos\left(\frac{\pi x}{4}\right) \cdot \frac{\pi}{4}\right] =$$

$$= -\frac{4}{\pi}\cos\left(\frac{\pi x}{4}\right) + x \sin\left(\frac{\pi x}{4}\right) + \frac{4}{\pi}\cos\left(\frac{\pi x}{4}\right)$$

$$= x \sin\left(\frac{\pi x}{4}\right) \blacktriangleleft$$

2) Calculate the volume of the solid generated by rotating the shaded square region around the y – axis.



$$dV = 2\pi x\,dx \cdot h = 2\pi x\,dx \cdot a =$$

$$= 2\pi a x\,dx$$

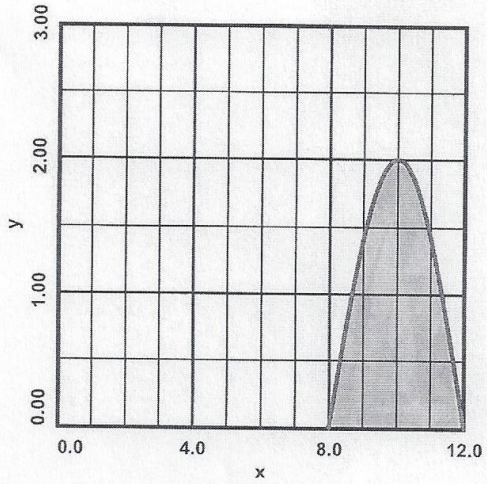
$$V = \int dV = 2\pi a \int_{R-\frac{a}{2}}^{R+\frac{a}{2}} x\,dx = 2\pi a \left[\frac{x^2}{2}\right]_{R-\frac{a}{2}}^{R+\frac{a}{2}} =$$

$$= \pi a \left[2Ra\right] = 2\pi R a^2 \blacktriangleleft$$

3) Calculate the volume of the solid generated by rotating the shaded area under

$$y = 2\sin\left(\frac{\pi x}{4}\right) \text{ on } x \in [8, 12]$$

around the y – axis. You will need to use the integral from problem **1**.



$$dV = 2\pi x\,dx \cdot h = 2\pi x\,dx \cdot 2\sin\left(\frac{\pi x}{4}\right)$$

$$= 4\pi x \sin\left(\frac{\pi x}{4}\right) dx$$

$$V = \int dV = 4\pi \int_{8}^{12} x \sin\left(\frac{\pi x}{4}\right) dx =$$

$$= \left[-16x\cos\left(\frac{\pi x}{4}\right) + \frac{64}{\pi}\sin\left(\frac{\pi x}{4}\right)\right]_{8}^{12} =$$

$$= 320 \blacktriangleleft$$