AP CALCULUS AB HOMEWORK #14

8. Pyramids, Cones and Frusta

Supplemental Problems
1

8.3. Prismatic Cross-Sections

Section 8.3 Exercises, pg. 414
1(a,b)[1], 3, 41

8.3. Solids of Revolution

Section 8.3 Exercises, pg. 414
7, 10[2,3], 11, 13, 24[3], 31(a), 33(b), 51

Supplemental Problems
2, **3**, **4**

8.3. Cylindrical Shells

Section 8.3 Exercises, pg. 414
36(a,c), 40

Chapter 8 Review Exercises, pg. 438
21(a)

Notes:

1. Do the integrals to calculate the volumes.

2. To do the integral, use the identity

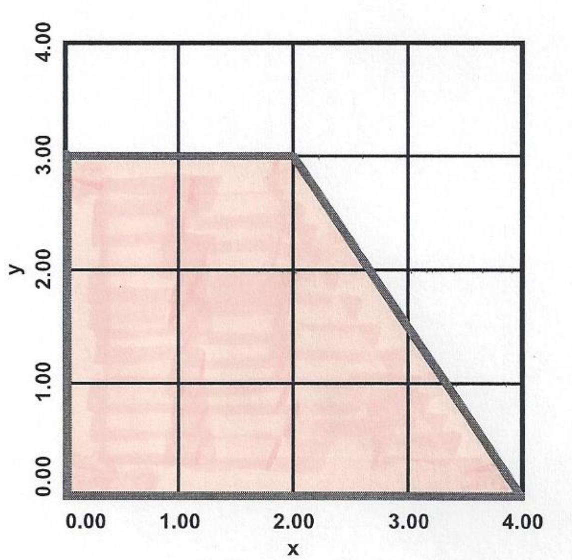
$$\sin 2x = 2\sin x \cos x \,.$$

3. To do the integral, use the identity

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)\,.$$

Supplemental Problems:

1) a) Calculate the volume of the solid generated by revolving the shaded area about the y – axis, *i.e.*, calculate the volume of the frustum so-generated. Use a cylindrical disk as the differential of volume $\mathrm{d}V$.



b) Check your answer to part **a** by using the formula derived in lecture, *viz.*,

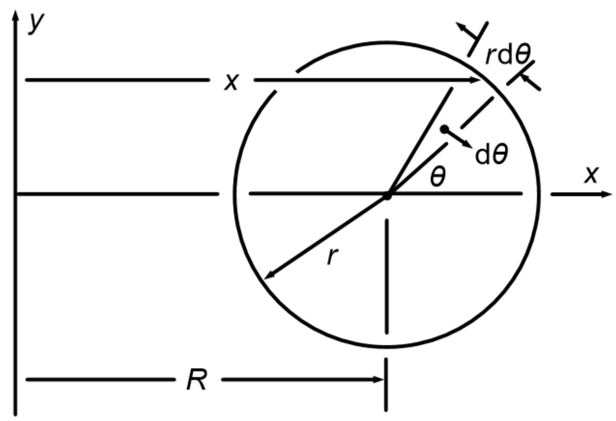
$$V = \frac{\pi(r_1^3 - r_2^3)h}{3(r_1 - r_2)}\,.$$

2) Given the area in the first quadrant bounded by the curves $y = \sin x$, $y = \cos x$ and the y – axis, calculate the volume of the solid obtained by rotating this area about the x – axis. Use an annular disk as the differential of volume. To do the resulting integral, use the identity $\cos^2 x - \sin^2 x = \cos 2x$.

3) *Surface Area of a Torus.* Revolving the circle shown around the y – axis generates a shape called a torus, which looks like a donut. When the arc length $r\mathrm{d}\theta$ is revolved around the y – axis, a differential of surface area $\mathrm{d}S$ is created. Since, on the differential scale, everything is linear (or flat), the differential of surface area is

$$\mathrm{d}S = (\text{length}) \cdot (\text{width}) = (2\pi x) \cdot (rd\theta) \,.$$

Express x in the expression for $\mathrm{d}S$ in terms of R, r and θ, and then integrate with respect to θ to derive the formula for the surface area of a torus.



4) *Volume of a Torus.* When the circle shown is revolved around the y – axis, once again, a torus is generated. To calculate the volume of the torus, use the differential of volume shown, which when revolved around the y – axis, is a toroidal shell. The volume $\mathrm{d}V$ of the shell is

$$dV = (\text{surface area of a torus}) \cdot \mathrm{d}\rho \,.$$

Use your formula for the surface area from problem **2** in the expression for $\mathrm{d}V$, and then integrate it with respect to ρ to derive the formula for the volume of a torus.