Supplemental

8. Pyramids, Cones and Frusta

(1)
(a) The right boundary is $y=-\frac{3}{2}x+6$, or $x=-\frac{2}{3}y+4$, $dV=\pi x^2\,dy=$

$=\pi\left(-\frac{2}{3}y+4\right)^2 dy=\pi\left(\frac{4}{9}y^2-\frac{16}{3}y+16\right)dy$, $V=\int dV=\pi\int_0^3\left(\frac{4}{9}y^2-\frac{16}{3}y+16\right)dy=$

$=\pi\left[\frac{4}{27}y^3-\frac{8}{3}y^2+16y\right]_0^3=28\pi$

(b) $r_1=4$, $r_2=2$, $h=3 \Rightarrow V=\frac{\pi(r_1^3-r_2^3)h}{3(r_1-r_2)}=\frac{\pi(64-8)\cancel{3}}{\cancel{3}(4-2)}=28\pi$

pg. 414 8.3. Prismatic Cross-Sections

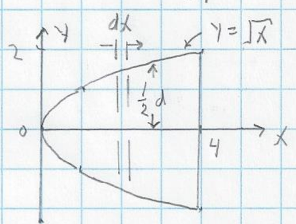
(1)
(a) $dV=\pi y^2\,dx$, $x^2+y^2=1$, $y^2=1-x^2$, $dV=\pi(1-x^2)\,dx$, $V=\pi\int_{-1}^{1}(1-x^2)\,dx=$

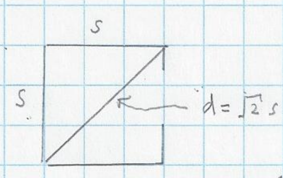
$=\pi\left[x-\frac{x^3}{3}\right]_{-1}^{1}=\frac{4}{3}\pi$

(b) $dV=s^2\,dx$, $\frac{1}{2}s=y$, $s=2y$, $s^2=4y^2=4(1-x^2)$, $dV=4(1-x^2)\,dx$,

$V=\int dV=4\int_{-1}^{1}(1-x^2)\,dx=4\left[\frac{4}{3}\right]=\frac{16}{3}=5\frac{1}{3}$

(3)



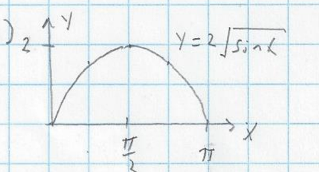


$dV=s^2\,dx$, $\frac{1}{2}d=\sqrt{x}=\frac{1}{2}(\sqrt{2}s)$,

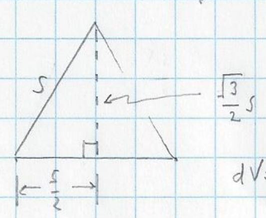
$\frac{1}{2}s^2=x$, $s^2=2x$, $dV=2x\,dx$

$V=\int dV=2\int_0^4 x\,dx=\cancel{2}\left[\frac{x^2}{\cancel{2}}\right]_0^4=16$

(4)



(a)



area $=\frac{1}{2}\cdot s\cdot\frac{\sqrt{3}}{2}s=\frac{\sqrt{3}}{4}s^2$

$dV=\frac{\sqrt{3}}{4}s^2\,dx=\frac{\sqrt{3}}{4}y^2\,dx=\sqrt{3}\sin x\,dx$

$V=\int dV=\sqrt{3}\int_0^{\pi}\sin x\,dx=\sqrt{3}\left[-\cos x\right]_0^{\pi}=2\sqrt{3}$ (b) $dV=s^2\,dx=y^2\,dx=4\sin x\,dx$

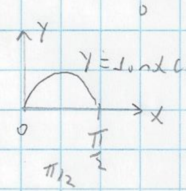
$V=\int dV=4\int_0^{\pi}\sin x\,dx=4[2]=8$

8.3. Solids of Revolution

(7) $x+2y=2$, $y=-\frac{1}{2}x+1$, $dV=\pi y^2 dx = \pi\left(-\frac{1}{2}x+1\right)^2 dx = \pi\left(\frac{1}{4}x^2-x+1\right)dx$

$V=\int dV = \pi\int_0^2\left(\frac{1}{4}x^2-x+1\right)dx = \pi\left[\frac{1}{12}x^3-\frac{1}{2}x^2+x\right]_0^2 = \frac{2}{3}\pi$ ◄

(10)

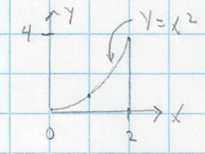


$dV=\pi y^2 dx = \pi\cdot\frac{1}{4}\sin^2 2x\cdot dx$,

$\sin^2 2x = \frac{1}{2}(1-\cos 4x)$, $dV=\frac{\pi}{8}(1-\cos 4x)dx$, $V=\int dV =$

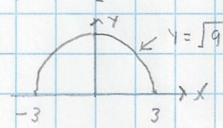
$= \frac{\pi}{8}\int_0^{\pi/2}(1-\cos 4x)dx = \frac{\pi}{8}\left[x-\frac{1}{4}\sin 4x\right]_0^{\pi/2} = \frac{\pi}{8}\left[\frac{\pi}{2}\right] = \frac{1}{16}\pi^2$ ◄

(11)



$dV=\pi y^2 dx = \pi x^4 dx$, $V=\int dV = \pi\int_0^2 x^4 dx = \pi\left[\frac{x^5}{5}\right]_0^2 = \frac{32}{5}\pi$ ◄

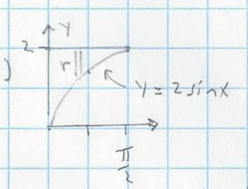
(13)



$dV=\pi y^2 dx = \pi(9-x^2)dx$, $V=\int dV = \pi\int_{-3}^{3}(9-x^2)dx =$

$= \pi\left[9x-\frac{x^3}{3}\right]_{-3}^{3} = 36\pi$ ◄

(24)



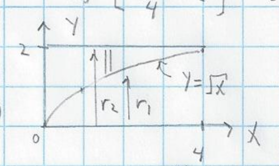
$dV=\pi r^2 dx = \pi(2-2\sin x)^2 dx = 4\pi(1-\sin x)^2 dx =$

$= 4\pi(1-2\sin x+\sin^2 x)dx = 4\pi\left(\frac{3}{2}-2\sin x-\frac{1}{2}\cos 2x\right)dx$

$V=\int dV = 4\pi\int_0^{\pi/2}\left(\frac{3}{2}-2\sin x-\frac{1}{2}\cos 2x\right)dx = 4\pi\left[\frac{3}{2}x+2\cos x-\frac{1}{4}\sin 2x\right]_0^{\pi/2} =$

$= 4\pi\left[\frac{3\pi}{4}-2\right] = 3\pi^2-8\pi$ ◄

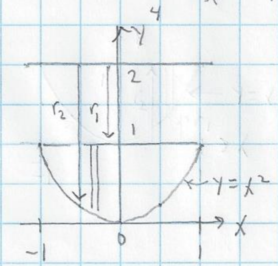
(31)
(a)



$dV=(\pi r_2^2-\pi r_1^2)dx = \pi(r_2^2-r_1^2)dx = \pi(4-x)dx$,

$V=\int dV = \pi\int_0^4(4-x)dx = \pi\left[4x-\frac{x^2}{2}\right]_0^4 = 8\pi$ ◄

(33)
(b)



$dV=(\pi r_2^2-\pi r_1^2)dx = \pi(r_2^2-r_1^2)dx$, $r_1=1$, $r_1^2=1$,

$r_2=2-x^2$, $r_2^2=4-4x^2+x^4$, $r_2^2-r_1^2=3-4x^2+x^4$

$V=\int dV = \pi\int_{-1}^{1}(3-4x^2+x^4)dx = \pi\left[3x-\frac{4}{3}x^3+\frac{x^5}{5}\right]_{-1}^{1} = \frac{56}{15}\pi$ ◄

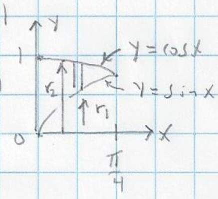
(51) (a) $dV = \pi y^2 dx$, $y = \frac{x}{12}\sqrt{36 - x^2}$, $y^2 = \frac{x^2}{144}(36 - x^2) = \frac{x^2}{4} - \frac{x^4}{144} = \frac{1}{4}\left(x^2 - \frac{x^4}{36}\right)$

$dV = \frac{\pi}{4}\left(x^2 - \frac{x^4}{36}\right)dx$, $V = \frac{\pi}{4}\int_0^6 \left(x^2 - \frac{x^4}{36}\right)dx = \frac{\pi}{4}\left[\frac{x^3}{3} - \frac{x^5}{180}\right]_0^6 = \frac{\pi}{4}\left[\frac{144}{5}\right] = \frac{36}{5}\pi \text{ cm}^3$ ◂

(b) Weight $= \left(\frac{36\pi}{5}\text{ cm}^3\right)\left(\frac{8.5 g}{\text{cm}^3}\right) = 192 g$ ◂

Supplemental

(2)



$dV = \pi(r_2^2 - r_1^2)dx = \pi(\cos^2 x - \sin^2 x)dx = \pi \cos 2x\, dx$

$V = \pi\int_0^{\pi/4} \cos 2x\, dx = \pi\left[\frac{1}{2}\sin 2x\right]_0^{\pi/4} = \frac{\pi}{2}$ ◂

(3) $dS = (2\pi x)\cdot(r\,d\theta)$, $x = R + r\cos\theta$, $dS = 2\pi(R + r\cos\theta)r\,d\theta = 2\pi r(R + r\cos\theta)d\theta$,

$S = \int dS = 2\pi r\int_0^{2\pi}(R + r\cos\theta)d\theta = 2\pi r\left[R\int_0^{2\pi} d\theta + r\int_0^{2\pi}\cos\theta\, d\theta\right] = 2\pi r\left[2\pi R + 0\right] = 4\pi^2 R r$ ◂

(4) $dV = (4\pi^2 R\rho)d\rho$, $V = 4\pi^2 R\int_0^r \rho\, d\rho = 4\pi^2 R\left[\frac{\rho^2}{2}\right]_0^r = 2\pi^2 R r^2$ ◂

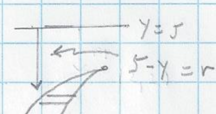
8.3. Cylindrical Shells

Pg. 414

(36) (a) $dV = 2\pi y\, dy \cdot w = 2\pi y\, dy \cdot \left[\frac{y^2}{2} - \left(\frac{y^4}{4} - \frac{y^2}{2}\right)\right] = 2\pi\left(y^3 - \frac{y^5}{4}\right)dy$, $V = \int dV =$

$= 2\pi\int_0^2 \left(y^3 - \frac{y^5}{4}\right)dy = 2\pi\left[\frac{1}{4}y^4 - \frac{1}{24}y^6\right]_0^2 = 2\pi\left[\frac{4}{3}\right] = \frac{8\pi}{3}$ ◂

(c)



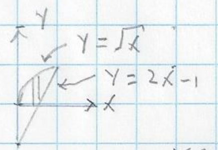
$dV = 2\pi r\, dy \cdot w = 2\pi(5 - y)dy \cdot \left(y^3 - \frac{y^5}{4}\right) =$

$= 2\pi\left(\frac{1}{4}y^6 - \frac{5}{4}y^5 - y^4 + 5y^3\right)dy$, $V = \int dV =$

$= 2\pi\int_0^2 \left(\frac{1}{4}y^6 - \frac{5}{4}y^5 - y^4 + 5y^3\right)dy = 2\pi\left[\frac{1}{28}y^7 - \frac{5}{24}y^6 - \frac{1}{5}y^5 + \frac{5}{4}y^4\right]_0^2 =$

$= 2\pi\left[\frac{508}{105}\right] = \frac{1016\pi}{105}$ ◂

(40)



$$dV = 2\pi x\, dx \cdot h = 2\pi x\, dx \left[\sqrt{x} - (2x-1)\right] =$$

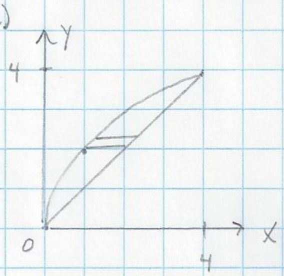
$$= 2\pi\left(x^{3/2} - 2x^2 + x\right) dx, \quad V = \int dV = 2\pi \int_0^1 \left(x^{3/2} - 2x^2 + x\right) dx =$$

$$= 2\pi \left[\frac{2}{5}x^{5/2} - \frac{2}{3}x^3 + \frac{x^2}{2}\right]_0^1 = 2\pi\left[\frac{7}{30}\right] = \frac{7\pi}{15}$$

pg. 438

(21) $y^2 = 4x$, $y = 2\sqrt{x}$, $x = \frac{1}{4}y^2$

(a)



$$dV = 2\pi y\, dy \cdot w = 2\pi y\, dy \left(y - \frac{1}{4}y^2\right) = 2\pi\left(y^2 - \frac{1}{4}y^3\right) dy$$

$$V = \int dV = 2\pi \int_0^4 \left(y^2 - \frac{1}{4}y^3\right) dy = 2\pi\left[\frac{y^3}{3} - \frac{y^4}{16}\right]_0^4 = 2\pi\left[\frac{16}{3}\right] =$$

$$= \frac{32\pi}{3}$$