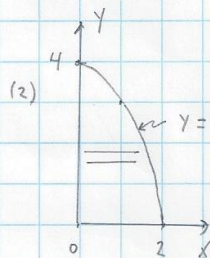


$$dV = s^2 dx, \quad \sqrt{2}s = y, \quad 2s^2 = y^2, \quad s^2 = \frac{1}{2}y^2$$

$$dV = \frac{1}{2}y^2 dx = \frac{1}{2}(-x^2 + 4x)^2 dx = \frac{1}{2}(x^4 - 8x^3 + 16x^2) dx$$

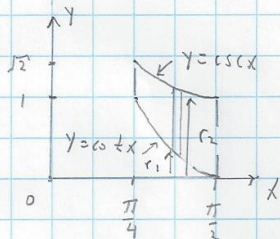
$$V = \int dV = \frac{1}{2} \int_0^4 (x^4 - 8x^3 + 16x^2) dx = \frac{1}{2} \left[\frac{x^5}{5} - 2x^4 + \frac{16}{3}x^3 \right]_0^4 = \frac{256}{15} = 17 \frac{1}{15}$$



$$y = -x^2 + 4, \quad 4 - y = x^2, \quad dV = \pi x^2 dy = \pi(4 - y) dy$$

$$V = \int dV = \pi \int_0^4 (4 - y) dy = \pi \left[4y - \frac{y^2}{2} \right]_0^4 = 8\pi$$

(3)

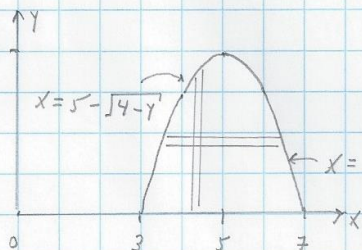


$$1 + \cot^2 x = \csc^2 x, \quad dV = \pi(r_2^2 - r_1^2) dx = \pi(\csc^2 x - \cot^2 x) dx = \pi dx$$

$$V = \int dV = \pi \int_{\pi/4}^{\pi/2} dx = \frac{\pi^2}{4}$$

(4)

(a)



$$y = 4 - (x-5)^2, \quad y = -x^2 + 10x - 21$$

$$y - 4 = -(x-5)^2, \quad (x-5)^2 = 4 - y, \quad x = 5 \pm \sqrt{4-y}$$

$$\text{cylindrical shell} \Rightarrow dV = 2\pi y \cdot dy \cdot w =$$

$$= 2\pi y \cdot dy \cdot (5 + \sqrt{4-y} - (5 - \sqrt{4-y})) =$$

$$= 4\pi y \sqrt{4-y} dy$$

$$V = \int dV = 4\pi \int_0^4 y \sqrt{4-y} dy = 4\pi \int_4^0 (4-u) \sqrt{u} \cdot -du =$$

$$u = 4-y, \quad y = 4-u, \quad dy = -du, \quad y=0 \rightarrow u=4, \quad y=4 \rightarrow u=0$$

$$= 4\pi \int_0^4 (4u^{1/2} - u^{3/2}) du = 4\pi \left[\frac{8}{3} u^{3/2} - \frac{2}{5} u^{5/2} \right]_0^4 = \frac{512}{15} \pi$$

(b) $dV = \pi y^2 dx = \pi (-x^2 + 10x - 21)^2 dx = \pi (x^4 - 20x^3 + 142x^2 - 420x + 441) dx$

Quiz #14 Study Guide

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$$V = \int dV = \pi \int_3^7 (x^4 - 20x^3 + 142x^2 - 420x + 441) dx = \pi \left[\frac{x^5}{5} - 5x^4 + \frac{142}{3}x^3 - 210x^2 + 441x \right]_3^7 = \frac{512}{15} \pi$$