AP CALCULUS AB — SPRING FINAL STUDY GUIDE

Problems **1** and **2** concern estimating the integral

$$\int_0^4 \left(4 - \frac{1}{4}x^2\right) \mathrm{d}x$$

using rectangles.

1) Estimate the integral using LRAM with four equal subdivisions.

2) Estimate the integral using RRAM with four equal subdivisions.

3) For

$$f(x) = \begin{cases} -\frac{1}{2}x + 3 & , \quad 0 \le x \le 6 \\ \sqrt{9 - (x-9)^2} & , \quad 6 < x \le 12 \end{cases},$$

evaluate

$$\int_0^{12} f(x)\, \mathrm{d}x$$

by using geometry.

4) Given that

$$\int_3^{10} f(x)\, \mathrm{d}x = 14\,, \int_7^{10} f(x)\, \mathrm{d}x = -3$$

and

$$\int_3^7 g(x)\, \mathrm{d}x = 9\,,$$

evaluate

$$\int_3^7 [\, 4f(x) + 5g(x)\,]\, \mathrm{d}x\,.$$

5) Evaluate

$$\int \left(\frac{91 \cdot \sqrt[4]{x^3}}{4} - \frac{104}{5 \cdot \sqrt[5]{x^{18}}} \right) \mathrm{d}x\,.$$

6) Evaluate

$$\int_0^{\pi/6} \sec^2 x\ \mathrm{d}x\,.$$

7) Evaluate

$$\int_0^4 \left(\, 4 - \frac{1}{4}x^2 \,\right) \mathrm{d}x\,.$$

8)

$$\frac{\mathrm{d}}{\mathrm{d}x} \int_{17}^{x} \sec^7 t\, \mathrm{d}t = ?$$

9)

$$\frac{\mathrm{d}}{\mathrm{d}x} \int_{x^6}^{x^8} \tan^4 t\ \mathrm{d}t = ?$$

10) For $f(x) = -x^2 + 4x$, calculate the average value of $f(x)$, *i.e.*, $\bar{f}$, over $x \in [0\,, 4]$.

11) For $f(x)$ as in problem **10**, find the values of $c \in [0\,, 4]$ which are guaranteed to exist via the Mean Value Theorem for Definite Integrals.

Problems **12** and **13** concern the function

$$f'(x) = \begin{cases} -\frac{10}{9}x + \frac{10}{3} & , \quad 0 \le x \le 2 \\ -\frac{64}{81}x + \frac{320}{81} & , \quad 2 < x \end{cases}.$$

12) Calculate

$$f(x) = \int_0^x f'(t)\, \mathrm{d}t\,.$$

13) Find the coordinates of the absolute maximum of $y = f(x)$.

14) For $y = f(x)$ as defined by the table

x	0	1	3	6
$f(x)$	2	3	6	5

estimate

$$\int_0^6 f(x)\, \mathrm{d}x$$

by using trapezoids. *Be careful!* The subdivision sizes are not equal.

15) For $y = f(x)$ as defined by the table

x	0	1	2	3	4	5	6
$f(x)$	25	32	27	16	5	0	7

estimate

$$\int_0^6 f(x)\,\mathrm{d}x$$

by using Simpson's Rule with three equal subdivisions.

16) Solve

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 18x + 4$$

for $y = y(x)$ subject to the initial conditions $y'(0) = 8$ and $y(0) = 7$.

17) For $x \in [-4\,,4]$ and $y \in [-4\,,4]$, draw the slope field of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x + y\,.$$

18) Solve

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{9x}{16y}$$

for $y = y(x)$ valid in the first quadrant subject to the initial condition $y(0) = 3$.

19) Evaluate

$$\int x\sqrt{16 - x^2}\,\mathrm{d}x\,.$$

20) Evaluate

$$\int \sec^2 x \tan x\;\mathrm{d}x\,.$$

21) Evaluate

$$\int_0^4 x\sqrt{16 - x^2}\,\mathrm{d}x\,.$$

22) Evaluate

$$\int_0^{\pi/3} \sec^2 x \tan x\;\mathrm{d}x\,.$$

23) After 500 years, 30% of a sample of a radioactive isotope decays. After how many years will 70% of the original sample be decayed?

24) Mom takes a meatloaf out of the oven and puts it on the kitchen counter. The temperature of the kitchen is 72°F. After 20 minutes, the temperature of the meatloaf is 180°F. Twenty-five minutes after that, its temperature is 105°F. What was the temperature of the meatloaf when it was first taken out of the oven?

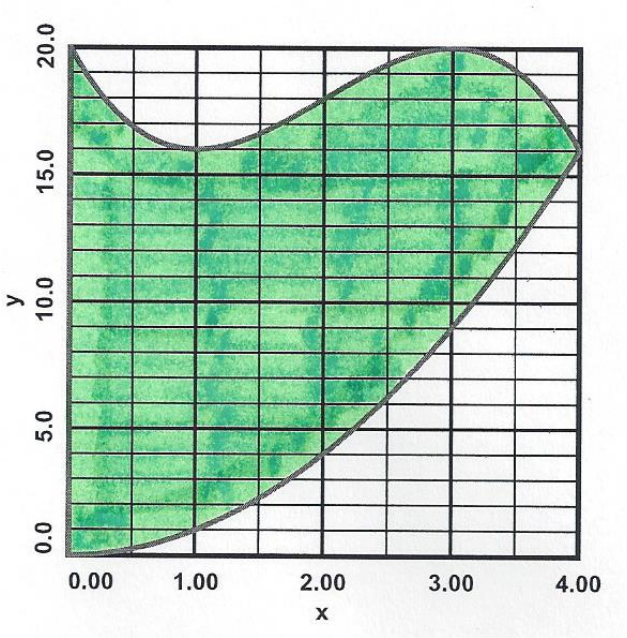
25)

$$\lim_{x\to 0}\frac{\cos x - 1}{x} = ?$$

26)

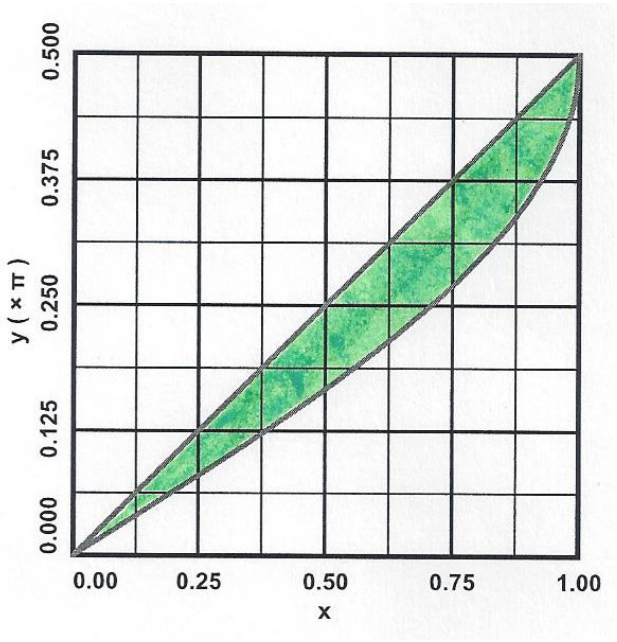
$$\lim_{x\to 0}\frac{\sin 4x}{\tan x} = ?$$

27) Calculate the area of the shaded region shown, which is bounded by the y – axis and the curves $y = -x^3 + 6x^2 - 9x + 20$ and $y = x^2$.



28) Calculate the area of the shaded region shown, which is bounded by the curves

$y = \frac{\pi}{2}x$ and $y = \sin^{-1} x$.



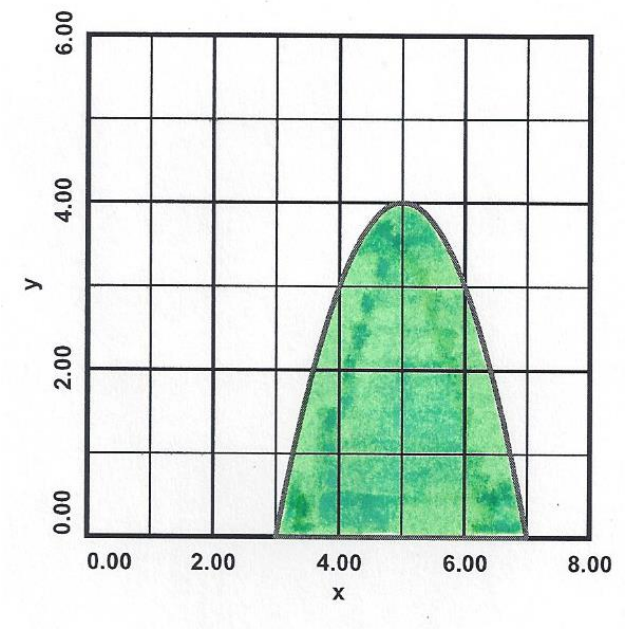
29) Calculate the volume of the solid, whose cross-sections perpendicular to the x – axis, are equilateral triangles, with their bases running from $y = -\sqrt{x}$ to $y = \sqrt{x}$ on $x \in [0\,,4]$.

30) For the area bounded by the x – axis and

$y = -\frac{1}{2}x^2 + 2$ on $x \in [-2\,,2]$,

calculate the volume of the solid generated by revolving this area about the x – axis.

Problems **31** and **32** concern the shaded area bounded by the x – axis and $y = 4 - (x-5)^2$ for $x \in [3\,,7]$ as shown.



In particular, these two problems concern calculating the volume of the solid generated by revolving this area about the y – axis.

31) Calculate the volume by using a cylindrical shell as the differential of volume.

32) Calculate the volume by using an annular disk as the differential of volume.