Spring Final Study Guide

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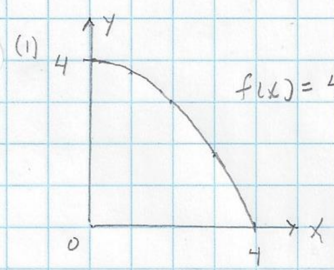


$f(x) = 4 - \frac{1}{4}x^2$

LRAM $\Delta x = 1$

$\text{area} \approx [f(0) + f(1) + f(2) + f(3)]\Delta x = 12.5$ ◄

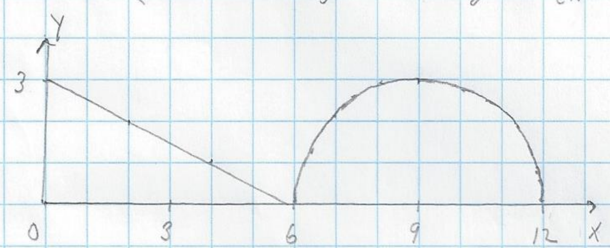
(2) RRAM $\Delta x = 1$

$\text{area} \approx [f(1) + f(2) + f(3) + f(4)]\Delta x = 8.5$ ◄

(3)

$$f(x) = \begin{cases} -\frac{1}{2}x + 3, & 0 \le x \le 6 \\ \sqrt{9-(x-9)^2}, & 6 < x \le 12 \end{cases}$$

$y = \sqrt{9-(x-9)^2}$, $y^2 = 9-(x-9)^2$, $(x-9)^2 + y^2 = 3^2$



$\int_0^{12} f(x)\,dx = \frac{1}{2}bh + \frac{1}{2}\pi r^2 =$

$= \frac{1}{2}(6)(3) + \frac{1}{2}\pi \cdot 3^2 = 9\left(1 + \frac{\pi}{2}\right)$ ◄

(4) $\int_3^{10} f(x)\,dx = 14$, $\int_7^{10} f(x)\,dx = -3$, $\int_3^{7} g(x)\,dx = 9 \Rightarrow \int_3^{10} f(x)\,dx = \int_3^{7} f(x)\,dx + \int_7^{10} f(x)\,dx$,

$14 = \int_3^7 f(x)\,dx - 3$, $\int_3^7 f(x)\,dx = 17$, $\int_3^7 [4f(x) + 5g(x)]\,dx = 4(17) + 5(9) = 113$ ◄

(5) $\int \left(\frac{91 \cdot \sqrt[4]{x^3}}{4} - \frac{104}{5 \cdot \sqrt[5]{x^{18}}} \right) dx = 13 \cdot \sqrt[4]{x^7} + \frac{8}{\sqrt[5]{x^{13}}} + k$ ◄

$f'(x) = \frac{91}{4}x^{3/4} - \frac{104}{5}x^{-18/5} \Rightarrow f(x) = \frac{91}{4} \cdot \frac{x^{7/4}}{7/4} - \frac{104}{5} \cdot \frac{x^{-13/5}}{-13/5} = 13x^{7/4} + 8x^{-13/5}$

(6) $\int_0^{\frac{\pi}{6}} \sec^2 x\,dx = \left[\tan x\right]_0^{\frac{\pi}{6}} = \frac{\sqrt{3}}{3}$ ◄

(7) $\int_0^4 \left(4 - \frac{1}{4}x^2\right) dx = \left[4x - \frac{1}{12}x^3\right]_0^4 = 10\frac{2}{3}$ ◄

(8) $\frac{d}{dx}\int_{17}^{x} \sec^7 t\,dt = \sec^7 x$ ◄

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(9) $\dfrac{d}{dx}\displaystyle\int_{a(x)}^{b(x)} f(t)\,dt = f(b)\dfrac{db}{dx} - f(a)\dfrac{da}{dx}$, $\dfrac{d}{dx}\displaystyle\int_{x^6}^{x^8} \tan^4 t\,dt =$

$= \tan^4(x^8)\cdot 8x^7 - \tan^4(x^6)\cdot 6x^5 = 8x^7\tan^4(x^8) - 6x^5\tan^4(x^6)$ ◄

(10) $f(x) = -x^2+4x$, $x \in [0,4]$, $\bar{f} = \dfrac{1}{4}\displaystyle\int_0^4 (-x^2+4x)\,dx = \dfrac{1}{4}\left[-\dfrac{x^3}{3}+2x^2\right]_0^4 = \dfrac{8}{3}$ ◄

(11) $-c^2+4c = \dfrac{8}{3}$, $c^2-4c+\dfrac{8}{3}=0$, $3c^2-12c+8=0$, $c = \dfrac{12 \pm \sqrt{12^2-4(3)(8)}}{2(3)} =$

$= \dfrac{12\pm\sqrt{48}}{6} = \dfrac{12\pm 4\sqrt{3}}{6} = 2 \pm \dfrac{2\sqrt{3}}{3}$ ◄

(12) $f'(x) = \begin{cases} -\dfrac{10}{9}x + \dfrac{10}{3}, & 0 \le x \le 2 \\ -\dfrac{64}{81}x + \dfrac{320}{81}, & 2 < x \end{cases}$, $f(x) = \displaystyle\int_0^x f'(t)\,dt$, $0 \le x \le 2$: $f(x) = -\dfrac{5}{9}x^2 + \dfrac{10}{3}x$,

$f(2) = \dfrac{40}{9}$, $2<x$: $f(x) = f(2) + \displaystyle\int_2^x \left(-\dfrac{64}{81}t + \dfrac{320}{81}\right)dt = \dfrac{40}{9} + \left[-\dfrac{32}{81}t^2 + \dfrac{320}{81}t\right]_2^x =$

$= \dfrac{40}{9} - \dfrac{32}{81}x^2 + \dfrac{320}{81}x - \dfrac{521}{81} = -\dfrac{32}{81}x^2 + \dfrac{320}{81}x - \dfrac{152}{81}$ $f(x) = \begin{cases} -\dfrac{5}{9}x^2 + \dfrac{10}{3}x, & 0 \le x \le 2 \\ -\dfrac{32}{81}x^2 + \dfrac{320}{81}x - \dfrac{152}{81}, & 2 < x \end{cases}$ ◄

(13) $0 \le x \le 2$: $f'(x) = 0 \Rightarrow x = 3$ (out of range) , $2 < x$: $f'(x) = 0 \Rightarrow x = 5$

$\Rightarrow (5, 8)$ is abs. max ◄ $f''(5) = -\dfrac{64}{81}$

kink: $f'(2^-) = \dfrac{10}{9} = 1\dfrac{1}{9}$, $f'(2^+) = 2\dfrac{10}{27}$ — not an extremum

(14)

x	0	1	3	6
$f(x)$	2	3	6	5

$\displaystyle\int_0^6 f(x)\,dx \approx \frac{1}{2}(2+3)\cdot 1 + \frac{1}{2}(3+6)\cdot 2 + \frac{1}{2}(6+5)\cdot 3 = 28$ ◄

(15)

x	0	1	2	3	4	5	6
$f(x)$	25	32	27	16	5	0	7

3 subdivisions $\Delta x = 2$

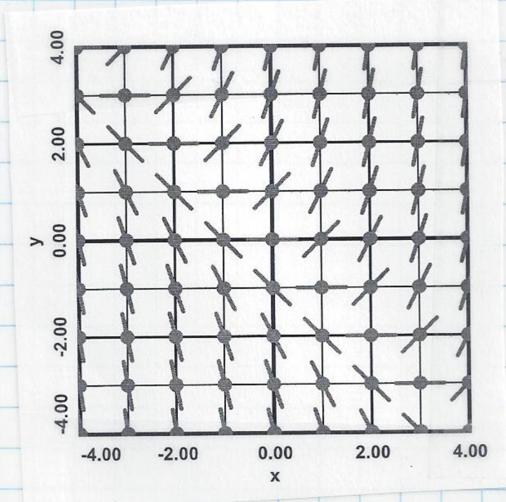
$\displaystyle\int_0^6 f(x)\,dx \approx \frac{\Delta x}{6}\left(25 + 4(32) + 2(27) + 4(16) + 2(5) + 4(0) + 7\right) = 96$ ◄

(16) $\frac{d^2y}{dx^2} = 18x+4$, $y'(0)=8$, $y(0)=7$, $dy' = (18x+4)\,dx$, $\int_8^{y'} dz = \int_0^x (18t+4)\,dt$,

$y'-8 = 9x^2+4x$, $\frac{dy}{dx} = 9x^2+4x+8$, $dy = (9x^2+4x+8)\,dx$,

$\int_7^y dz = \int_0^x (9t^2+4t+8)\,dt$, $y-7 = 3x^3+2x^2+8x$, $y = 3x^3+2x^2+8x+7$

(17) $\frac{dy}{dx} = x+y$



(18) $\frac{dy}{dx} = -\frac{9x}{16y}$, $y(0)=3$, $16y\,dy = -9x\,dx$, $16\int_3^y z\,dz = -9\int_0^x t\,dt$, $16\left[\frac{z^2}{2}\right]_3^y = -9\left[\frac{t^2}{2}\right]_0^x$

$16(y^2-9) = -9x^2$, $16y^2 - 144 = -9x^2$, $16y^2 = 144-9x^2 = 9(16-x^2)$,

$y^2 = \frac{9}{16}(16-x^2)$, $y = \frac{3}{4}\sqrt{16-x^2}$

(19) $\int x\sqrt{16-x^2}\,dx = \int \cancel{x}\sqrt{u}\cdot -\frac{du}{2\cancel{x}} = -\frac{1}{2}\int \sqrt{u}\,du = -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}+k =$

$\boxed{u = 16-x^2,\ \frac{du}{dx} = -2x,\ dx = -\frac{du}{2x}}$

$= -\frac{1}{3}\sqrt{(16-x^2)^3}+k$

(20) $\int \sec^2 x \tan x\,dx = \int \cancel{\sec^2 x}\,u\,\frac{du}{\cancel{\sec^2 x}} = \int u\,du = \frac{1}{2}u^2+k =$

$\boxed{u = \tan x,\ \frac{du}{dx} = \sec^2 x,\ dx = \frac{du}{\sec^2 x}}$

$= \frac{1}{2}\tan^2 x + k$

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(21) $\int_0^4 x\sqrt{16-x^2}\,dx = -\frac{1}{2}\int_{16}^{0}\sqrt{u}\,du = \frac{1}{2}\int_0^{16}\sqrt{u}\,du = \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_0^{16} = 21\frac{1}{3}$ ◄

$\boxed{\text{prob (19)} \Rightarrow x=0 \to u=16,\ x=4 \to u=0}$

(22) $\int_0^{\pi/3} \sec^2 x \tan x\,dx = \int_0^{\sqrt{3}} u\,du = \left[\frac{u^2}{2}\right]_0^{\sqrt{3}} = \frac{3}{2}$ ◄

$\boxed{\text{prob (20)} \Rightarrow x=0 \to u=0,\ x=\pi/3 \to u=\sqrt{3}}$

(23) $C = C_0 e^{-kt}$, $0.7C_0 = C_0 e^{-500k}$, $-500k = \ln 0.7$, $k = -\frac{1}{500}\ln 0.7 = 7.133\times 10^{-4}/\text{yr}$,

$0.3\cancel{C_0} = \cancel{C_0}e^{-kt}$, $-kt = \ln 0.3$, $t = -\frac{1}{k}\ln 0.3$, $t = 1688$ yrs ◄

(24) $T = T_\infty + (T_0 - T_\infty)e^{-kt}$, $T = 72 + (T_0 - 72)e^{-kt}$

$$\left.\begin{array}{ll} 180 = 72 + (T_0-72)e^{-20k}, & 108 = (T_0-72)e^{-20k} \\ 105 = 72 + (T_0-72)e^{-45k}, & 33 = (T_0-72)e^{-45k} \end{array}\right\} (\div) \Rightarrow \frac{108}{33} = e^{25k},$$

$\ln\left(\frac{108}{33}\right) = 25k$, $k = \frac{1}{25}\ln\left(\frac{108}{33}\right) = 0.047425/\text{min}$, $108e^{20k} = T_0 - 72$,

$T_0 = 72 + 108e^{20k} = 350.8°F$ ◄

(25) $\lim_{x\to 0}\frac{\cos x - 1}{x} = \frac{0}{0} = \lim_{x\to 0}\frac{-\sin x}{1} = \frac{0}{1} = 0$ ◄

(26) $\lim_{x\to 0}\frac{\sin 4x}{\tan x} = \frac{0}{0} = \lim_{x\to 0}\frac{4\cos 4x}{\sec^2 x} = \frac{4\cdot 1}{1} = 4$ ◄

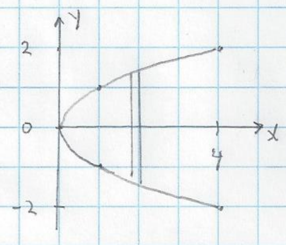
(27) $dA = (-x^3+6x^2-9x+20-x^2)\,dx = (-x^3+5x^2-9x+20)\,dx$

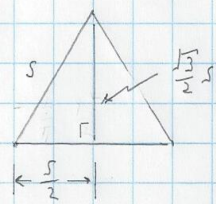
$A = \int dA = \int_0^4 (-x^3+5x^2-9x+20)\,dx = \left[-\frac{x^4}{4}+\frac{5}{3}x^3-\frac{9}{2}x^2+20x\right]_0^4 = 50\frac{2}{3}$ ◄

(28) $y=\sin^{-1}x \to x=\sin y$, $y=\frac{\pi}{2}x \to x=\frac{2}{\pi}y$, $dA=\left(\sin y-\frac{2}{\pi}y\right)dy$,

$$A=\int dA=\int_0^{\pi/2}\left(\sin y-\frac{2}{\pi}y\right)dy=\left[-\cos y-\frac{1}{\pi}y^2\right]_0^{\pi/2}=1-\frac{\pi}{4} \blacktriangleleft$$

(29)





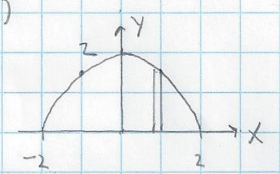
$$\text{area}=\frac{1}{2}\cdot s\cdot\frac{\sqrt{3}}{2}s=\frac{\sqrt{3}}{4}s^2$$

$$\frac{1}{2}s=\sqrt{x},\quad s=2\sqrt{x},\quad s^2=4x$$

$$dA=\frac{\sqrt{3}}{4}s^2dx=\frac{\sqrt{3}}{4}4x\,dx=\sqrt{3}\,x\,dx,$$

$$A=\int dA=\sqrt{3}\int_0^4 x\,dx=\sqrt{3}\left[\frac{x^2}{2}\right]_0^4=8\sqrt{3} \blacktriangleleft$$

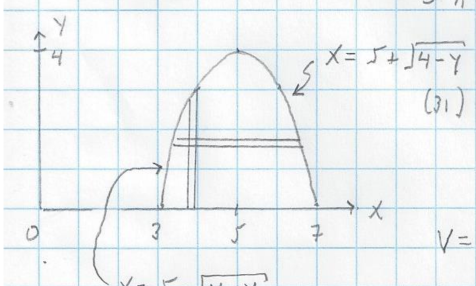
(30)



$$y=-\frac{1}{2}x^2+2,\quad y^2=\frac{1}{4}x^4-2x^2+4,\quad dV=\pi y^2dx=$$

$$=\pi\left(\frac{1}{4}x^4-2x^2+4\right)dx,\quad V=\int dV=\pi\int_{-2}^{2}\left(\frac{1}{4}x^4-2x^2+4\right)dx=$$

$$=\pi\left[\frac{1}{20}x^5-\frac{2}{3}x^3+4x\right]_{-2}^{2}=8\tfrac{8}{15}\pi \blacktriangleleft$$



(31) $y-4=-(x-5)^2$, $y-4=-(x^2-10x+25)=-x^2+10x-25$,

$$y=-x^2+10x-21,\quad dV=2\pi x\,dx\,y=2\pi(-x^3+10x^2-21x)dx$$

$$V=\int dV=2\pi\int_3^7(-x^3+10x^2-21x)dx=2\pi\left[-\frac{x^4}{4}+\frac{10}{3}x^3-\frac{21}{2}x^2\right]_3^7=$$

$$=106\tfrac{2}{3}\pi \blacktriangleleft$$

(32) $4-y=(x-5)^2$, $x-5=\pm\sqrt{4-y}$, $x=5\pm\sqrt{4-y}$, $dV=\pi(r_2^2-r_1^2)dy$

$r_2=5+\sqrt{4-y}$, $r_2^2=25+10\sqrt{4-y}+4-y=29-y+10\sqrt{4-y}$ $\quad r_2^2-r_1^2=20\sqrt{4-y}$

$r_1=5-\sqrt{4-y}$, $r_1^2=25-10\sqrt{4-y}+4-y=29-y-10\sqrt{4-y}$ $\quad dV=20\pi\sqrt{4-y}\,dy$

$$V=\int dV=20\pi\int_0^4\sqrt{4-y}\,dy=20\pi\int_4^0\sqrt{u}\cdot -du=20\pi\int_0^4\sqrt{u}\,du=20\pi\left[\frac{2}{3}u^{3/2}\right]_0^4=$$

$\boxed{u=4-y,\ dy=-du,\ y=0\to u=4,\ y=4\to u=0}$ $\quad =20\pi\left[\frac{16}{3}\right]=106\frac{2}{3}\pi \blacktriangleleft$